

Understanding of the freeze-out in ultra-relativistic heavy-ion collisions

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Abstract I discuss the quantities and effects important for the freeze-out and outline a formalism for the description of continuous decoupling of particles from the fireball. Then, I present a calculation of the scattering rates of pions at various temperatures and argue that it is important to take continuous particle decoupling into account when modelling the collision dynamics.

Key words continuous freeze-out • escape probability

Hadronic single-particle spectra and correlations carry information about the freeze-out state of the fireball which results from the collective expansion of the strongly interacting matter. Often, the freeze-out state is being modelled in framework of the Cooper-Frye [7] mechanism where all particles – regardless of their identity and/or momentum – are emitted from a single sharp three-dimensional freeze-out hypersurface. This is the case for most hydrodynamic simulations. A question appears: does the assumption of a common sharp three-dimensional freeze-out still provide reliable approximation of the real process?

In case of a sharp freeze-out, the freeze-out hypersurface is usually characterised by some prescription. There were some attempts to identify a universal freeze-out criterion [1, 8], i.e., a condition which determines the freeze-out hypersurface for heavy-ion collisions of any size and at any ultrarelativistic energy. In case of the continuous gradual decoupling, there is no hypersurface to be determined and the concept of a universal freeze-out criterion is not applicable. I will focus on gradual decoupling since the sharp freeze-out can be defined as its limiting case.

Let me focus on the mechanism of freeze-out and identify the important effects and quantities. Freeze-out occurs when scattering ceases. It has been suggested that the mean-free path is the relevant quantity to look at [1, 11]. Here, densities of the individual species are weighted with cross-sections for scattering on them. An example: density of nucleons is much more important for pion scattering rate than the density of pions, because the πN cross-section is bigger than the one for $\pi\pi$ scattering. The CERES collaboration argued that the universal pion mean-free path at freeze-out should be something of the order of 1 fm (maybe 2–3 fm) [1].

On the first sight this is rather surprising, because this length is much shorter than the size of the system. So far, however, we did not mention the expansion and the decrease of the density due to it. In rough terms, freeze-

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out occurs when the rate of the density decrease becomes comparable or larger than the scattering rate [6]. The picture is the following: at the moment of scattering, the particle has some finite mean-free path λ and thus would be expected to scatter after some time. However, if the density drops fast enough, after passing the length λ , our particle may find itself in an environment of very low density such that no other scattering occurs.

In [15], we used the formalism of escape probabilities (conceived earlier by other authors, e.g. [9, 14]) and focused on the dependence of the escape probability on temperature and the momentum of pions. By using this formalism one qualitatively goes beyond probing the mean-free path and the rate of density decrease; particles now decouple from a finite four-volume. Sharp freeze-out is realised as a limiting case of this more general case, when the escape probabilities of all particles change from 0 to 1 in a very narrow space-time region.

The escape probability can be determined as

$$(1) \quad P(x, \tau, p) = \exp\left(-\int_{\tau}^{\infty} d\bar{\tau} R(x - v\bar{\tau}, p)\right)$$

where R is the scattering rate. The opacity integral in this equation is evaluated along the expected trajectory of a particle with momentum p and the position characterised by x and τ if it moved straight. The integrated scattering rate gives the average number of collisions the particle would suffer on this trajectory. Assumptions about the chemical composition of the medium are included in calculation of R , while the density decrease rate determines its time dependence and thus the value of the resulting integral.

As an example of the fireball expansion, we assumed in [15] a particle with $p_t = 0$ in the centre of the fireball, which does not move transversely, but just waits until the surrounding matter decays. Then, we only need a prescription for the time dependence of R for which we take an ansatz:

$$(2) \quad R(\tau) = R_0 \left(\frac{\tau_0}{\tau}\right)^\alpha$$

where $\alpha > 1$ and R_0 is the scattering rate at time τ_0 . This corresponds to a fireball expanding longitudinally in a boost-invariant manner and transversely with the following dependence of the transverse flow rapidity on radial coordinate r :

$$(3) \quad \eta_t(r, \tau) = \frac{\alpha - 1}{2} \frac{r}{\tau}.$$

The escape probability in this toy model then reads

$$(4) \quad P(p_t = 0, r = 0) = \exp\left(-\frac{R_0 \tau_0}{\alpha - 1}\right) = \exp\left(-\frac{R_0}{2\eta_t(r, \tau_0)/r}\right).$$

Obviously, larger R_0 leads to smaller escape probability, whereas stronger expansion is encoded in larger α and/or η_t and increases the value of P .

We calculated the scattering rate of negative pions in thermal hadronic gas as

$$(5) \quad R(p) = \sum_i \int d^3k \rho_i(k) \sigma_i'(s) |v_\pi - v_i|^*.$$

Here, $\rho_i(k)$ is the density of particles on which the pion scatters, $\sigma_i(s)$ is the collinear cross-section, and $|v_\pi - v_i|^*$ is the relative velocity of the pion to the other scattering partner in the CMS of the pair. We integrate over the momenta of the scatterers and sum over following species as scattering partners: $i = \pi, N, \bar{N}, K, \rho, \Delta, \bar{\Delta}$. For the cross-section we use a parameterisation of hadron scattering via resonances (see [5, 15]). For the temperatures, we assumed values of 90, 100 and 120 MeV. We estimated the chemical potentials in such a way that we reproduced data on pion phase-space density at the SPS [4, 10] and RHIC [12] and the ratios of dN/dy of different species at midrapidity [2, 3, 13].

In Fig. 1 I compare pion scattering rates in hadronic gases corresponding to those at the SPS and at RHIC. The contribution from nucleons and antinucleons to the total

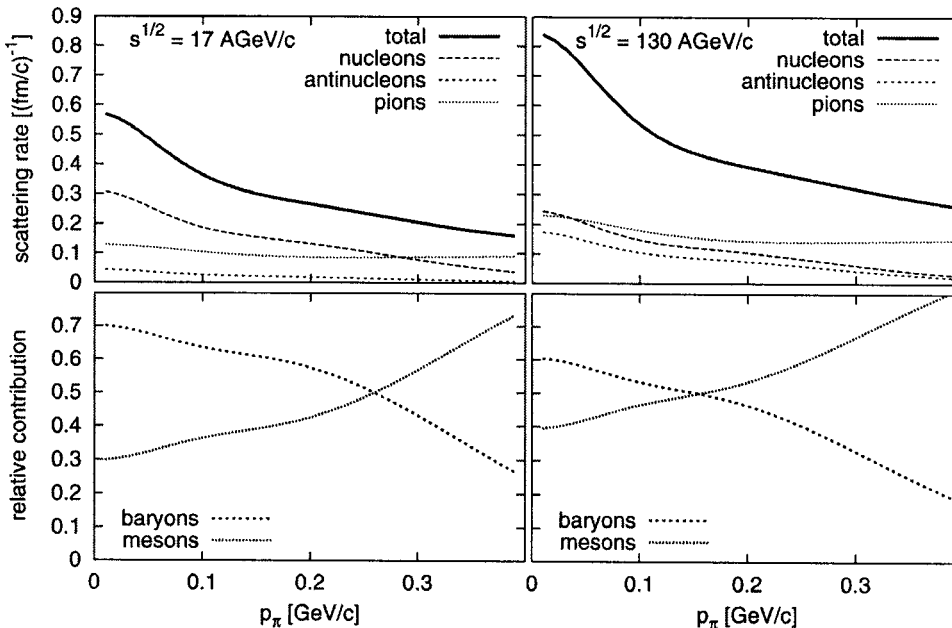


Fig. 1. Scattering rate as a function of pion momentum with respect to the hadronic medium with $T = 100$ MeV. Results are obtained for SPS (left) and RHIC (right). Contributions to the scattering rate from scattering on pions, nucleons and antinucleons are indicated. The two lower panels show the baryonic and mesonic relative contributions.

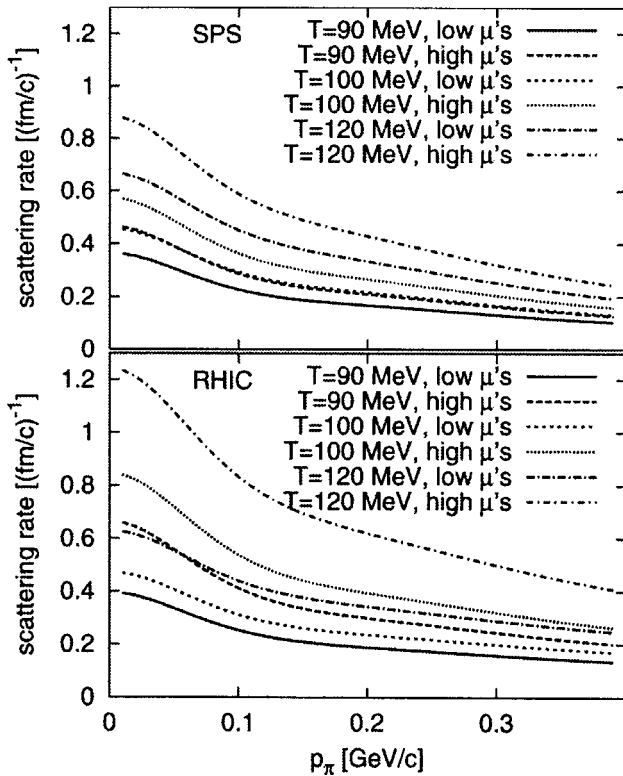


Fig. 2. The pion scattering rates as functions of momentum with respect to the medium, calculated for different temperatures and sets of chemical potentials allowed by data. Details can be found in the original paper [15].

scattering rate increases little when moving from SPS to the higher energy system. A higher phase-space density of pions at RHIC [12] leads to larger pion contribution, but the total scattering rate is not dominated by pions and, therefore, does not change much. Quantitatively, about 10% relative contribution is shifted from baryons to mesons when going from SPS to RHIC.

A summary of results is plotted in Fig. 2. The scattering rate drops with temperature. When we use eq. (4) and a realistic estimate of the transverse rapidity gradient [16], reasonable escape probabilities ($\sim 30\text{--}50\%$) are obtained for $T \leq 100$ MeV. Note that this was inferred for a particle with $p_t = 0$ in centre of the fireball.

One can see that regardless of the assumed temperature and chemical potentials higher momentum particles always have smaller scattering rate and thus decouple easier. This suggests that the freeze-out can be sequential: high momenta first, low momenta later. A sequential freeze-out might be an additional cause of the M_t dependence of the HBT radii, if high- p_t pions decouple earlier and from a smaller fireball than the low- p_t ones. Such a scenario, however, must be studied in more detail. Nevertheless, it represents an

interesting alternative to the blast-wave model, which seems to have problems in reproducing data on HBT radii.

Due to the momentum dependence of the scattering rate, collapsing the whole decoupling four-volume into a sharp three-dimensional freeze-out hypersurface seems an unreliable approximation.

References

1. Adamová D, Agakichiev G, Appelshäuser H *et al.* (2003) Universal pion freeze-out in heavy ion collisions. *Phys Rev Lett* 90:022301
2. Adcox K, Adler SS, Ajitanand NN *et al.* (2002) Centrality dependence of $\pi^{+/-}$, $K^{+/-}$, p , and \bar{p} production from $\sqrt{s_{NN}} = 130$ GeV Au+Au collisions at RHIC. *Phys Rev Lett* 88:242301
3. Afanasiev SV, Anticic T, Barna D *et al.* (2002) Energy dependence of pion and kaon production in central Pb+Pb collisions. *Phys Rev C* 66:054902
4. Aggarwal MM, Angelis ALS, Antonenko V *et al.* (2003) One, two and three particle distributions from 158 AGeV/c central Pb+Pb collisions. *Phys Rev C* 67:014906
5. Bass SA, Belkacem M, Bleicher M *et al.* (1998) Microscopic models for ultrarelativistic heavy-ion collisions. *Prog Part Nucl Phys* 41:225–370
6. Bondorf JP, Garpman SIA, Zimányi J (1978) A simple analytic hydrodynamic model for expanding fireballs. *Nucl Phys A* 296:320–332
7. Cooper F, Frye G (1974) Single-particle distribution in the hydrodynamic and statistical thermodynamic models of multiparticle production. *Phys Rev D* 10:186–189
8. Ferenc D, Heinz U, Tomášik B, Wiedemann UA, Cramer JG (1999) Universal pion freeze-out phase-space density. *Phys Lett B* 457:374–378
9. Grassi F, Hama Y, Kodama T (1995) Continuous particle emission: a probe of thermalized matter evolution? *Phys Lett B* 355:9–14
10. Murray M, Holzer B (2001) (Anti)proton and pion source sizes and phase space densities in heavy ion collisions. *Phys Rev C* 63:054901
11. Nagamiya S (1982) Role of mean free paths of product particles in high energy nucleus nucleus collisions. *Phys Rev Lett* 49:1383–1386
12. Ray RL (for the STAR Collaboration) (2003) Correlations, fluctuations, and flow measurements from the STAR experiment. *Nucl Phys A* 715:45c–54c
13. Siklér F, Bächler J, Barna D *et al.* (1999) Hadron production in nuclear collisions from the NA49 experiment at 158 GeV/c. *Nucl Phys A* 661:45c–54c
14. Sinyukov YuM, Akkelin SV, Hama Y (2002) On freeze-out problem in hydro-kinetic approach to A+A collisions. *Phys Rev Lett* 89:052301
15. Tomášik B, Wiedemann UA (2003) Universal freeze-out mechanism and phase-space density in ultrarelativistic heavy-ion collisions. *Phys Rev C* 68:034905
16. Tomášik B, Wiedemann UA, Heinz U (2003) Reconstructing the freeze-out state in Pb+Pb collisions at 158 AGeV/c. *Heavy Ion Phys* 17:105–143