

# Buda-Lund hydro model and the elliptic flow at RHIC

Máté Csanád,  
Tamás Csörgő,  
Bengt Lörstad,  
András Ster

**Abstract** The ellipsoidally symmetric Buda-Lund hydrodynamic model describes naturally the transverse momentum and the pseudorapidity dependence of the elliptic flow in Au+Au collisions at  $\sqrt{s_{NN}} = 130$  and 200 GeV. The result confirms the indication of quark deconfinement in Au+Au collisions at RHIC, obtained from Buda-Lund hydro model fits to combined spectra and HBT radii of BRAHMS, PHOBOS, PHENIX and STAR.

**Key words** elliptic flow • hydrodynamical models • quark-gluon plasma

## Introduction

PHENIX, PHOBOS and STAR experiments at RHIC produced a wealth of information on the asymmetry of the particle spectra with respect to the reaction plane [1–4, 6, 7], characterized by the second harmonic moment of the transverse momentum distribution, denoted by  $v_2$ . It is measured as a function of the transverse mass and particle-type at midrapidity as well as a function of the pseudorapidity  $\eta = 0.5 \log[(|p| + p_z)/(|p| - p_z)]$ .

The PHOBOS collaboration found [6], that  $v_2(\eta)$  is a strongly decreasing function of  $|\eta|$ , which implies that the concept of boost-invariance, suggested by Bjorken in Ref. [9], cannot be applied to characterize the hadronic final state of Au+Au collisions at RHIC.

We summarize here a successful attempt to describe the pseudorapidity dependence of the elliptic flow  $v_2(\eta)$  at RHIC, for more details see Ref. [13]. Our tool is the Buda-Lund hydrodynamic model [21, 22], which we extended in Ref. [13] from axial to ellipsoidal symmetry.

## Buda-Lund hydro model for ellipsoidal expansions

Based on the success of the Buda-Lund hydro model to describe Au+Au collisions at RHIC [14, 24], Pb+Pb collisions at CERN SPS [28] and h+p reactions at CERN SPS [5, 17], we describe the emission function in the core-halo picture [23], and assume that the core evolves in a hydrodynamical manner:

$$(1) \quad S_c(x, p) d^4x = \frac{g}{(2\pi)^3} \frac{p^\mu d^4\Sigma_\mu(x)}{B(x, p) + s_q}$$

where  $g$  is the degeneracy factor ( $g = 1$  for identified

M. Csanád  
Department of Atomic Physics,  
ELTE,  
Pázmány P. 1/a, H-1117 Budapest, Hungary

T. Csörgő✉, A. Ster  
Research Institute for Particle Physics  
of the Hungarian Academy of Sciences  
(MTA KFKI RMKI),  
H-1525 Budapest 114, P. O. Box 49, Hungary,  
Tel.: +36 1 3922222/ 3397, Fax: +36 1 3922598,  
e-mail: csorgo@sunserv.kfki.hu

B. Lörstad  
Department of Physics,  
University of Lund,  
S-22362 Lund, Sweden

Received: 26 January 2004

pseudoscalar mesons,  $g = 2$  for identified spin =  $1/2$  baryons), and  $p^\mu d^4\Sigma_\mu(x)$  is a generalized Cooper-Frye term, describing the flux of particles through a distribution of layers of freeze-out hypersurfaces,  $B(x,p)$  is the (inverse) Boltzmann phase-space distribution, and the term  $s_q$  is determined by quantum statistics,  $s_q = 0, -1,$  and  $+1$  for Boltzmann, Bose-Einstein and Fermi-Dirac distributions, respectively.

For a hydrodynamically expanding system, the (inverse) Boltzmann phase-space distribution is

$$(2) \quad B(x, p) = \exp\left(\frac{p^\nu u_\nu(x)}{T(x)} - \frac{\mu(x)}{T(x)}\right).$$

We will utilize some ansatz for the shape of the flow four-velocity,  $u_\nu(x)$ , chemical potential,  $\mu(x)$ , and temperature,  $T(x)$  distributions. Their form is determined with the help of recently found exact solutions of hydrodynamics, both in the relativistic [19, 20] and in the non-relativistic cases [15, 16, 25].

The generalized Cooper-Frye prefactor is determined from the assumption that the freeze-out happens, with probability  $H(\tau)d\tau$ , at a hypersurface characterized by  $\tau = \text{const}$  and that the proper-time measures the time elapsed in a fluid element that moves together with the fluid,  $d\tau = u^\mu(x)dx_\mu$ . We parameterize this hypersurface with the coordinates  $(r_x, r_y, r_z)$  and find that  $d^3\Sigma^\mu(x|\tau) = u^\mu(x)d^3x u^0(x)$ . Using  $\partial_r \tau|_r = u^0(x)$  we find that in this case the generalized Cooper-Frye prefactor is

$$(3) \quad p^\mu d^4\Sigma_\mu(x) = p^\mu u_\mu(x) H(\tau) d^4x.$$

This finding generalizes a result of Ref. [8, 12, 26, 27] from the case of a spherically symmetric Hubble flow to anisotropic, direction dependent Hubble flow distributions.

From the analysis of CERN SPS and RHIC data [14, 24, 28], we find that the proper-time distribution in heavy ion collisions is rather narrow, and  $H(\tau)$  can be well approximated with a Gaussian representation of the Dirac-delta distribution,

$$(4) \quad H(\tau) = \frac{1}{(2\pi\Delta\tau^2)^{1/2}} \exp\left(-\frac{(\tau - \tau_0)^2}{2\Delta\tau^2}\right)$$

with  $\Delta\tau \ll \tau_0$ .

We specify a fully scale invariant, relativistic form, which reproduces known non-relativistic hydrodynamic solutions too, in the limit when the expansion is non-relativistic. Both in the relativistic and the non-relativistic cases, the ellipsoidally symmetric, self-similarly expanding hydrodynamical solutions can be formulated in a simple manner, using a scaling variable  $s$  and a corresponding four-velocity distribution  $u^\mu$ , that satisfy

$$(5) \quad u^\mu \partial_\mu s = 0$$

which means that  $s$  is a good scaling variable if its co-moving derivative vanishes [19, 20].

It is convenient to introduce the dimensionless, generalized space-time rapidity variables  $(\eta_x, \eta_y, \eta_z)$ , defined by the identification of

$$(6) \quad \sinh \eta_x = r_x \frac{\dot{X}}{X}$$

similar equations hold for  $y$  and  $z$ . The characteristic sizes (for example, the lengths of the major axis of the expanding ellipsoid) are  $(X, Y, Z)$  that depend on proper-time  $\tau$  and their derivatives with respect to proper-time are denoted by  $(\dot{X}, \dot{Y}, \dot{Z})$ . Eq. (5) is satisfied by the choice of

$$(7) \quad s = \frac{\cosh \eta_x - 1}{\dot{X}_f^2} + \frac{\cosh \eta_y - 1}{\dot{Y}_f^2} + \frac{\cosh \eta_z - 1}{\dot{Z}_f^2}$$

$$(8) \quad u^\mu = (\gamma, \sinh \gamma_x, \sinh \gamma_y, \sinh \gamma_z)$$

and from here on  $(\dot{X}_f, \dot{Y}_f, \dot{Z}_f) = \{\dot{X}(\tau_0), \dot{Y}(\tau_0), \dot{Z}(\tau_0)\} = (\dot{X}_1, \dot{X}_2, \dot{X}_3)$ , assuming that the rate of expansion is constant in the narrow proper-time interval of the freeze-out process. The above form has the desired non-relativistic limit,

$$(9) \quad s \rightarrow \frac{r_x^2}{2X_f^2} + \frac{r_y^2}{2Y_f^2} + \frac{r_z^2}{2Z_f^2}$$

where again  $(X_f, Y_f, Z_f) = (X(\tau_0), Y(\tau_0), Z(\tau_0)) = (X_1, X_2, X_3)$ . From now on, we drop subscript  $f$ . The normalization condition of  $u^\mu(x)u_\mu(x) = 1$  yields the value of  $\gamma$ . For the fugacity distribution we assume a shape, that leads to Gaussian profile in the non-relativistic limit,

$$(10) \quad \frac{\mu(x)}{T(x)} = \frac{\mu_0}{T_0} - s$$

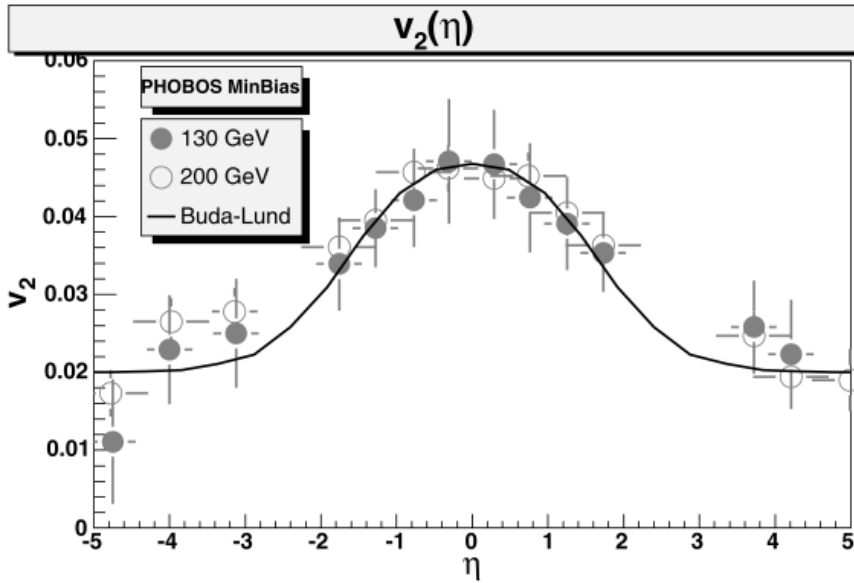
corresponding to the solution discussed in Refs. [15, 16, 18]. We assume that the temperature may depend on the position as well as on proper-time. We characterize the inverse temperature distribution similarly to the shape used in the axially symmetric model of Refs. [21, 22], and discussed in the exact hydro solutions of Refs. [15, 16].

$$(11) \quad \frac{1}{T(x)} = \frac{1}{T_0} \left(1 + \frac{T_0 - T_s}{T_s} s\right) \left(1 + \frac{T_0 - T_e}{T_e} \frac{(\tau - \tau_0)}{2\Delta\tau^2}\right)$$

where  $T_0$ ,  $T_s$  and  $T_e$  are the temperatures of the center, and the surface at the mean freeze-out time  $\tau_0$ , while  $T_e$  corresponds to the temperature of the center after most of the particle emission is over (cooling due to evaporation and expansion). Sudden emission implies  $T_e = T_0$  and  $\Delta\tau \rightarrow 0$ .

The observables can be calculated analytically from the Buda-Lund hydro model, using a saddle-point approximation in the integration. This approximation is exact both in the Gaussian and the non-relativistic limit, and if  $p^\nu u_\nu / T \gg 1$  at the point of maximal emittivity.

The results are summarized in Figs. 1 and 2. We find that a small asymmetry between the two transverse Hubble constants gives a natural explanation of the transverse momentum dependence of  $v_2$ . The parameters are taken from Buda-Lund hydro model fits in Refs. [14, 24], where the axially symmetric version of the model was utilized,



**Fig. 1.** Here we see the fit to the PHENIX  $v_2(p_t)$  data of identified particles [10, 11]. The parameter set is:  $T_0 = 210$  MeV,  $X_f = 0.57$ ,  $Y_f = 0.45$ ,  $Z_f = 2.4$ ,  $T_s = 105$  MeV,  $\tau_0 = 7$  fm/c,  $v = 0.09$ ,  $X_f = 8.6$  fm,  $Y_f = 105$  fm,  $Z_f = 17.5$  fm,  $\mu_{0,\pi} = 70$  MeV,  $\mu_{0,K} = 210$  MeV and  $\mu_{0,p} = 315$  MeV the masses are taken as their physical value.

although the mean transverse flow is reduced due to the less central nature of collisions studied in this study, which is a summary of Ref. [13]. Note that we did not yet fine-tune the ellipsoidal Buda-Lund hydro model to describe these  $v_2$  data, instead we searched the parameter set by hand. So at the moment we are not yet ready to report the best fit parameters and the error bars on the extracted parameter values. However, as indicated by Figs. 1 and 2, even this fit by eye method is successful in reproducing the data on elliptic flow at RHIC.

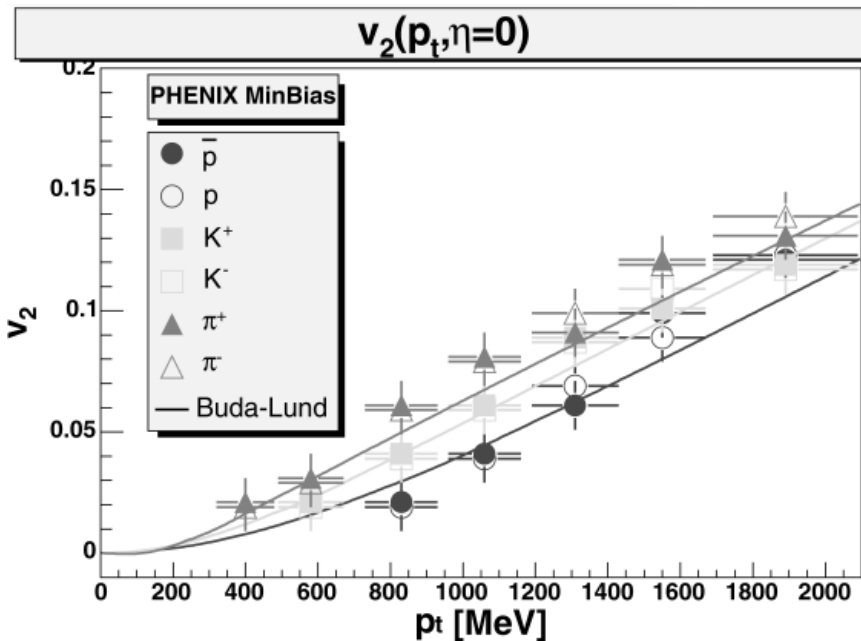
First we tuned the model to describe the  $p_t$  dependent elliptic flow of identified particles at midrapidity, as shown in Fig. 1. Then we calculated the value of the transverse momentum integrated  $v_2(\eta = 0)$  and found [13], that this value is below the published PHOBOS data point at midrapidity. We attribute the difference of 0.02 to a non-flow contribution [4, 10, 11]. The PHOBOS collaboration pointed out the possible existence of such a non-flow contribution

in their data in Ref. [6], as they did not utilize the fourth order cumulant measure of  $v_2$ .

We note that our presently best choice of parameter set correspond to a high,  $T_0 > T_c = 170$  MeV central temperature, with a cold surface temperature of  $T_s \approx 105$  MeV, see Figs. 1 and 2.

### Summary and conclusions

We have generalized the Buda-Lund hydro model to the case of ellipsoidally symmetric expanding fireballs. We kept the parameters as determined from fits to the single particle spectra and the two-particle Bose-Einstein correlation functions (HBT radii) [14, 24], and interpreted them as angular averages over the direction of the reaction plane. Then we observed that a small splitting between the expansion rates parallel and transverse to the direction of the impact par-



**Fig. 2.** This image shows the fit to the 130 GeV Au+Au and 200 GeV Au+Au  $v_2(\eta)$  data of PHOBOS [21, 23], with the ellipsoidal generalization of the Buda-Lund hydro model. Here we used the same parameter set as at Fig. 1, with pion mass and chemical potential, and a constant non-flow parameter of 0.02.

ameter, as well as a small zenithal tilt of the particle emitting source is sufficient to describe simultaneously the transverse momentum dependence of the collective flow of identified particles [2] as well as the pseudorapidity dependence of the collective flow [6, 7] at RHIC.

The results confirm the indication for quark deconfinement at RHIC found in Refs. [14, 24], based on the observation, that some of the particles are emitted from a region with higher than the critical temperature,  $T > T_c = 170$  MeV. The size of this volume is about  $750 \text{ fm}^3$ , corresponding to 1/8th of the total volume measured on the  $\tau = \tau_0$  main freeze-out hypersurface [13]. At the same time, this analysis indicates that the surface temperature is rather cold,  $T_s \approx 105$  MeV, so approximately 7/8 of the particles are emitted from a rather cold hadron gas. So the picture is similar to a fireball, which is heated from inside.

**Acknowledgment** T. Csörgő and M. Csanád would like to thank the Organizers for their kind hospitality and for their creating an inspiring and fruitful meeting. The support of the following grants are gratefully acknowledged: OTKA T038406, 33421, the OTKA-MTA-NSF grant INT0089462 and the exchange programme of the Hungarian and Polish Academy of Sciences.

## References

- Adcox K, Adler SS, Ajitanan NN *et al.* (2002) Flow measurements via two-particle azimuthal correlations in Au+Au collisions at  $s(\text{NN})^{1/2} = 130$  GeV. *Phys Rev Lett* 89:212301
- Adler C, Ahammed Z, Allgower C *et al.* (2001) Identified particle elliptic flow in Au+Au collisions at  $s(\text{NN})^{1/2} = 130$  GeV. *Phys Rev Lett* 87:182301
- Adler C, Ahammed Z, Allgower C *et al.* (2002) Elliptic flow from two- and four-particle correlations in Au+Au collisions at  $s(\text{NN})^{1/2} = 130$  GeV. *Phys Rev C* 66:034904
- Adler SS, Afanasiev S, Aidala C *et al.* (2003) Elliptic flow of identified hadrons in Au+Au collisions at  $s(\text{NN})^{1/2} = 200$  GeV. *Phys Rev Lett* 91:182301
- Agababyang NM, Atayang MR, Csörgő T *et al.* (1998) Estimation of hydrodynamical model parameters from the invariant spectrum and the Bose-Einstein correlations of pi- mesons produced in (pi+/K+)p interactions at 250 GeV/c. *Phys Lett B* 422:359–368
- Back BB, Baker MD, Barton DS *et al.* (2002) Pseudorapidity and centrality dependence of the collective flow of charged particles in Au+Au collisions at  $s(\text{NN})^{1/2} = 130$  GeV. *Phys Rev Lett* 89:222301
- Back BB, Baker MD, Barton DS *et al.* (2003) Flow and Bose-Einstein correlations in Au-Au collisions at RHIC. *Nucl Phys A* 715:611–614
- Baran A, Broniowski W, Florkowski W (2003) Thermal model at RHIC. II: Elliptic flow and HBT radii. *AIP Conf Proc* 660:185–195
- Bjorken JD (1983) Highly relativistic nucleus-nucleus collisions: The central rapidity region. *Phys Rev D* 27:140–151
- Borghini N, Dinh PM, Ollitrault JY (2001) A new method for measuring azimuthal distributions in nucleus-nucleus collisions. *Phys Rev C* 63:054906
- Borghini N, Dinh PM, Ollitrault JY (2001) Flow analysis from multiparticle azimuthal correlations. *Phys Rev C* 64:054901
- Broniowski W, Florkowski W (2001) Explanation of the RHIC p(T)-spectra in a thermal model with expansion. *Phys Rev Lett* 87:272302
- Csanád M, Csörgő T, Lörstad B (2003) Buda-Lund hydro model for ellipsoidally symmetric fireballs and the elliptic flow at RHIC. (nucl-th/0310040)
- Csanád M, Csörgő T, Lörstad B, Ster A (2004) An indication for deconfinement in Au+Au collisions at RHIC. *Acta Phys Pol B* 35:191–196, (nucl-th/0311102)
- Csörgő T (1998) Simple analytic solution of fireball hydrodynamics. (nucl-th/9809011)
- Csörgő T (2001) Simple solutions of fireball hydrodynamics for selfsimilar, ellipsoidal flows. (hep-ph/0111139)
- Csörgő T (2002) Particle interferometry from 40 MeV to 40 TeV. *Heavy Ion Phys* 15:1–80
- Csörgő T, Akkelin SV, Hama Y, Lukács B, Sinyukov YuM (2003) Observables and initial conditions for self-similar ellipsoidal flows. *Phys Rev C* 67:034904
- Csörgő T, Csernai L, Hama Y, Kodama T (2003) Simple solutions of relativistic hydrodynamics for systems with ellipsoidal symmetry. (nucl-th/0306004)
- Csörgő T, Grassi F, Hama Y, Kodama T (2003) Simple solutions of relativistic hydrodynamics for longitudinally and cylindrically expanding systems. *Phys Lett B* 565:107–115
- Csörgő T, Lörstad B (1995) Bose-Einstein correlations for expanding finite systems. *Nucl Phys A* 590:465c–468c
- Csörgő T, Lörstad B (1996) Bose-Einstein correlations for three-dimensionally expanding, cylindrically symmetric, finite systems. *Phys Rev C* 54:1390–1403
- Csörgő T, Lörstad B, Zimányi J (1996) Bose-Einstein correlations for systems with large halo. *Z Phys C* 71:491–497
- Csörgő T, Ster A (2003) The reconstructed final state of Au+Au collisions from PHENIX and STAR data at  $s^{1/2} = 130$  A GeV: Indication for quark deconfinement at RHIC. *Heavy Ion Phys* 17:295–312
- Csörgő T, Zimányi J (2003) Inflation of fireballs, the gluon wind and the homogeneity of the HBT radii at RHIC. *Heavy Ion Phys* 17:281–293
- Florkowski W, Broniowski W (2003) Thermal description of transverse momentum spectra at RHIC. *Nucl Phys A* 715:875–878
- Florkowski W, Broniowski W (2003) Thermal model for RHIC. I: Particle ratios and spectra. *AIP Conf Proc* 660:177–184
- Ster A, Csörgő T, Lörstad B (1999) The reconstructed final state of Pb+Pb 158 GeV/A reactions from spectra and correlation data of NA49, NA44 and WA98. *Nucl Phys A* 661:419c–422c