Nuclear data for the cyclotron production of $^{64}\text{Cu}$, $^{86}\text{Y}$, $^{76}\text{Br}$, $^{64}\text{Cu}$ and $^{43}\text{Sc}$ in PET imaging

**Abstract.** Positron emission tomography (PET) is a powerful diagnostic tool, which provides superior spatial resolution and an opportunity to obtain quantitative information concerning distribution of radioactivity in vivo. Most interesting positron emitters for the purpose of diagnosis are $^{64}\text{Cu}$, $^{124}\text{I}$, $^{18}\text{F}$, $^{86}\text{Y}$, $^{48}\text{V}$, $^{52}\text{Mn}$, $^{140}\text{Pr}$, $^{72}\text{As}$, $^{74}\text{As}$, $^{89}\text{Zr}$, $^{82}\text{Sr}$, $^{68}\text{Ga}$, $^{45}\text{Ti}$, $^{88}\text{Br}$ and $^{89}\text{Rb}$. Aim of the presented study is to compare the calculated cross sections of several radioisotopes of positron emitters as follows $^{86}\text{Y}$, $^{43}\text{Sc}$, $^{64}\text{Cu}$, $^{66}\text{Ga}$ and $^{76}\text{Br}$ with incident proton energy up to 30 MeV. In this work, excitation function of positron emitters via the $^{86}\text{Sr}(p,n)^{86}\text{Y}$, $^{43}\text{Ca}(p,n)^{43}\text{Sc}$, $^{66}\text{Zn}(p,n)^{66}\text{Ga}$, $^{40}\text{Ni}(p,n)^{40}\text{Cu}$ and $^{76}\text{Se}(p,n)^{76}\text{Br}$ reactions were calculated by ALICE/ASH 0.1 (GHD model and hybrid model) and TALYS-1.2 (equilibrium and pre-equilibrium) codes and compared to existing data. Requisite for optimal thicknesses of targets were obtained by the stopping and range of ions in matter (SRIM) code for each reaction.

**Key words:** production yields • excitation function • positron emitters • ALICE/ASH • TALYS-1.2
Material and methods

Calculation of excitation function

Excitation functions of $^{86}$Sr, $^{43}$Ca, $^{64}$Ni, $^{66}$Zn and $^{76}$Se + p reactions were calculated by using ALICE/ASH and TALYS-1.2 codes [24, 25, 27]. The codes were used simultaneously to increase the accuracy of calculations. An optimum energy range was determined and employed to avoid the formation of radionuclide impurities and decrease the excitation functions of inactive impurities as far as possible. To further achieve the aim, feasibility of the production of positron emitters via proton induced per low/medium energy accelerators was investigated.

Nuclear level densities

Investigation of nuclear level densities is of great interest in nuclear physics, since they are of importance both in developing a consistent theoretical description of the properties of excited nuclei and in calculating cross sections for nuclear reactions within the statistical model. Such calculations include investigations into nuclear synthesis in nuclear astrophysics. Since the Bethe model of 1936 some more or less successful phenomenological expressions based, e.g. on the Fermi-gas or the so-called back shifted Fermi-gas model were proposed to reproduce the existing data and predict the not yet measured cases. Theoretical calculations within the shell model and the Monte Carlo (MC) methods, which generally include pairing correlations, and take the influence of spin and parity into account, have been quite successful in this context [4, 8, 10, 34, 35, 56].

The ALICE/ASH code

The ALICE/ASH code is a modified and advanced version of the ALICE code [5]. The geometry dependent hybrid (GDH) model is used for the description of the preequilibrium particle emission. Intra-nuclear transition rates are calculated using the effective cross section of nucleon-nucleon interactions in nuclear matter. Corrections are made to the GDH model for the description of preequilibrium complex particle emission. The exciton coalescence model and the knock-out model are used for the description of the preequilibrium complex particle emission. The equilibrium emission of particles is described by the Weisskopf-Ewing model without detail consideration of angular momentum [6, 28, 48, 51].

The nuclear level density is taken in the form

$$\rho(U) = (U - \delta)^{-\frac{5}{4}} \exp(2\sqrt{\alpha(U - \delta)})$$

and the level density parameter is equal

$$a = A/y$$

where $y$ is a constant. The pairing correction, $\delta$ can be evaluated using different schemes depending upon the input parameter $MP$: “standard” shift ($MP = 3$)

$$\delta = 11/A^{1/2}$$

for even-even nuclei,

$$\delta = 0$$

for nuclei with odd $A$,

$$\delta = -11/A^{1/2}$$

for odd-odd nuclei

“backshift” ($MP = 1$)

$$\delta = 0$$

for even-even nuclei

$$\delta = -11/A^{1/2}$$

for nuclei with odd $A$

$$\delta = -22/A^{1/2}$$

for odd-odd nuclei

At excitation energies below 2 MeV, the level density is calculated by the “constant” temperature model.

Kataria-Ramamurthy Fermi gas model [21]

$$a = \alpha A + \beta_0 A^{2/3} + \beta_1 A^{2/3} S_A + \beta_2 A^{2/3} S_p$$

The parameter $a$ is on the basis of a number of model single-particle energy level schemes. Simple functional form for the parameter $a$ was proposed, taken into account the effect of the Fermi energy of nucleons.

The parameters have been obtained by least square fit to the ‘experimental’ values for spherical nuclides in the mass-region $40 < A < 210$ ($\alpha = 0.08$ MeV$^{-1}$, $\beta_0 = -0.12$ MeV$^{-1}$, $\beta_1 = 1.35$, $\beta_2 = 1.4$) [22, 23].

Fermi gas model of Ignatyuk, Smirenkin, Tishin with an energy dependent level density parameter

The nuclear level density is defined by the expression

$$\rho(U) = a^{1/4} (U - \delta)^{-\frac{5}{4}} \exp(2\sqrt{\alpha(U - \delta)})$$

The nuclear level density parameter is calculated as follows [27]

$$a(U) = \tilde{a} (1 + f(U) \delta W/U)$$

where $\delta W$ is the shell correction,

$$f(U) = 1 - \exp(-\gamma U)$$

$$\tilde{a} = A(\alpha + \beta A),$$

where $\alpha = 0.154$, $\beta = -6.3 \times 10^{-4}$ and $\gamma = 0.054$ MeV$^{-1}$. The pairing correction is

$$\delta = 24/A^{1/2}$$

for even-even nuclei

Fermi gas model with an energy independent level density parameter
(15) \( \delta = 12A^{1/2} \) for nuclei with odd \( A \)

(16) \( \delta = 0 \) for odd-odd nuclei

At excitation energies < 2 MeV, the level density is calculated using the “constant” temperature approach.

**Superfluid nuclear model**

The nuclear level density is calculated according to the generalized Superfluid Model [19]

\[
\rho(U) = \rho_{qp}(U') K_{vb}(U') K_{ao}(U')
\]

where \( \rho_{qp}(U') \) is the density of quasi-particle nuclear excitation, \( K_{vb}(U') \) and \( K_{ao}(U') \) are the vibrational and rotational enhancement factors at the effective energy of excitation \( U' \) calculated by Ignatyuk [17].

The nuclear level density parameters are calculated according to the expression [18].

\[
a(U) = \begin{cases} 
\hat{a}(1 + \delta W \varphi) U' - E_{\text{cond}} / (U' - E_{\text{cond}}), & U' > U_\sigma, \\
\hat{a}(U_\sigma), & U' \leq U_\sigma
\end{cases}
\]

where \( \delta W \) is the shell correction to the mass formula equal to the difference between experimental mass defect and the one calculated from the liquid drop model [33]

\[
\varphi(U) = 1 - \exp(-\gamma U), \quad \gamma = 0.4/A^{1/3} \text{MeV}^{-1}
\]

The asymptotic value of nuclear level parameter is equal to

\[
\hat{a} = A(0.073 + 0.115A^{-1/3})
\]

The effective energy of excitation \( U' \), the critical energy of the phase transition \( U_\sigma \) and the condensation energy \( E_{\text{cond}} \) are calculated as follows:

\[
U' = U - n\Delta_0
\]

\[
U_\sigma = 0.472 a(U_\sigma) \Delta_0^2 - n\Delta_0
\]

\[
E_{\text{cond}} = 0.152 a(U_\sigma) \Delta_0^2 - n\Delta_0
\]

The correlation function \( \Delta_0 \) is equal to

\[
\Delta_0 = 12A^{1/2}
\]

where \( n = 0 \) for even-even nuclei, \( n = 1 \) for nuclei with odd \( A \) value, \( n = 2 \) for odd-odd nuclei. The precompound emission was described using the GDH model [6, 26, 49].

**The TALYS 1.2 code**

TALYS 1.2 code is optimized for incident projectile energies, ranging from 1 keV up to 200 MeV on target nuclei with mass numbers between 12 and 339. It includes photon, neutron, proton, deuteron, triton, \(^3\)He, and \( \alpha \)-particles as both projectiles and ejectiles, and single-particle as well as multi-particle emissions and fission. All experimental information on nuclear masses, deformation, and low-lying states spectra is considered, whenever available and if not, various local and global input models have been incorporated to represent the nuclear structure properties, optical potentials, level densities, \( \gamma \)-ray strengths, and fission properties. The TALYS code was designed to calculate total and partial cross sections, residual and isomer production cross sections, discrete and continuum \( \gamma \)-ray production cross sections, energy spectra, angular distributions, double differential spectra, as well as recoil cross sections. The preequilibrium particle emission is described using the two-component exciton model. The model implements new expressions for internal transition rates and new parameterization of the average squared matrix element for the residual interaction obtained using the optical model potential [3, 25]. The phenomenological model is used for the description of the preequilibrium complex particle emission. The equilibrium particle emission is described using the Hauser-Feshbach model.

In statistical models for predicting cross sections, nuclear level densities are used at excitation energies where discrete level information is not available or incomplete. Several models use for the level density in TALYS, which range from phenomenological analytical expressions to tabulated level densities derived from microscopic models. To set the notation, first give some general definitions. The level density \( \rho(E_\gamma, J, \Pi) \) corresponds to the number of nuclear levels per MeV around an excitation energy \( E_\gamma \), for a certain spin \( J \) and parity \( \Pi \). The total level density \( \rho_{\text{tot}}(E_\gamma) \) corresponds to the total number of levels per MeV around \( E_\gamma \), and is obtained by summing the level density over spin and parity:

\[
\rho_{\text{tot}}(E_\gamma) = \sum_j \sum_\Pi \rho(E_\gamma, J, \Pi)
\]

The nuclear levels are degenerate in \( M \), the magnetic quantum number, which gives rise to the total state density \( \omega_{\text{tot}}(E_\gamma) \) which includes the \( 2J + 1 \) states for each level, i.e.

\[
\omega_{\text{tot}}(E_\gamma) = \sum_j \sum_\Pi (2J + 1) \rho(E_\gamma, J, \Pi)
\]

When level densities are given by analytical expressions they are usually factorized as follows

\[
\rho(E_\gamma, J, \Pi) = P(E_\gamma, J, \Pi) R(E_\gamma, J) \varphi_\gamma(E_\gamma)
\]

where \( P(E_\gamma, J, \Pi) \) is the parity distribution and \( R(E_\gamma, J) \) the spin distribution. In all but one level density model in TALYS, the parity equipartition is assumed, i.e. [25]

\[
P(E_\gamma, J, \Pi) = \frac{1}{2}
\]

**Calculation of the required thickness of target**

According to SRIM code the required thickness of target was calculated [57]. The physical thickness of the target layer is chosen in such a way that for a given beam/target angle geometry (90°) the incident beam be exited of target layer with predicted energy. To minimize thickness of the target layer, 6° geometry
is preferred; so the required layer thickness will be less with coefficient 0.1. The calculated thicknesses were shown for ideal reactions in Table 1.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Isotopic abundance (%)</th>
<th>Energy range (MeV)</th>
<th>Target thickness (μm)</th>
<th>Theoretical field (MBq/μA.h)</th>
<th>Q-value (MeV)</th>
<th>E-threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{86}$Sr(p,n)$^{86}$Y</td>
<td>9.86</td>
<td>13 → 18</td>
<td>73.45</td>
<td>793.90</td>
<td>−6.02</td>
<td>6.092</td>
</tr>
<tr>
<td>$^{76}$Se(p,n)$^{76}$Br</td>
<td>9.36</td>
<td>12 → 17</td>
<td>49</td>
<td>372.46</td>
<td>−5.74</td>
<td>6.048</td>
</tr>
<tr>
<td>$^{66}$Zn(p,n)$^{66}$Ga</td>
<td>27.9</td>
<td>9 → 15</td>
<td>61</td>
<td>446.53</td>
<td>−5.95</td>
<td>6.040</td>
</tr>
<tr>
<td>$^{43}$Ca(p,n)$^{43}$Sc</td>
<td>0.135</td>
<td>6 → 13</td>
<td>220</td>
<td>110.28</td>
<td>−3.003</td>
<td>3.073</td>
</tr>
<tr>
<td>$^{64}$Ni(p,n)$^{64}$Cu</td>
<td>0.926</td>
<td>8 → 13</td>
<td>101</td>
<td>68.31</td>
<td>−2.457</td>
<td>2.4961</td>
</tr>
</tbody>
</table>

The production yield can be calculated as below

$$ Y = \frac{N_L H}{M} \int_{E_0}^{E_l} \left( \frac{dE}{d(\rho x)} \right)^{-1} \sigma(E) dE $$

where $Y$ is the product activity (in Bq) of the product, $N_L$ is the Avogadro number, $H$ is the isotope abundance of the target nuclide, $M$ is the mass number of the target.

**Calculation of theoretical yield**

Enhance of the projectile energy, the beam current and the time of bombardment increase the production yield.

**Fig. 1.** Excitation function of $^{86}$Sr(p,n)$^{86}$Y reaction (a) by TALYS-1.2 code (b) via experimental data [11], level density options of ALICE/ASH 0.1 and TALYS-1.2 codes.
element, $\sigma(E)$ is the cross section at energy $E$, $I$ is the projectile current, $dE/d(x)$ is the stopping power, $\lambda$ is the decay constant of the product and $t$ is the time of irradiation. The production yields of positron emitters via different reactions were calculated using the Simpson numerical integral as of Eq. (29) (Table 1) [45].

**Results and discussion**

Excitation function of $^{86}$Sr(p,n)$^{86}$Y reaction

Excitation functions of the proton-induced reaction on strontium-86 were measured by ALICE/ASH and TALYS-1.2 codes and compared to existing data. Figure 1 shows a comparison between calculated cross section from options of ALICE-ASH code, TALYS-1.2 code, and the experimental data that have been studied by Levkovskij [30] and Rösch et al. [41]. As a result, experimental data by Levkovskij are lesser than those of Rösch et al. Also, there is a relatively good agreement between the experimental data by Levkovskij and the prediction of the excitation function made by ALICE/ASH and TALYS-1.2 codes. The evaluation of the acquired data showed that the best range of the energy is 13 to 18 MeV. According to SRIM code the required target thickness should be 73.45 μm.

Figure 1. Comparison of calculated cross sections from options of ALICE-ASH code, TALYS-1.2 code, and the experimental data that have been studied by Levkovskij [30] and Rösch et al. [41].

Excitation function of $^{76}$Se(p,n)$^{76}$Br reaction

Using the $^{76}$Se(p,n)$^{76}$Br reaction to produce $^{76}$Br, the best range of incident energy was assumed 17 to 12 MeV whose maximum cross section by ALICE/ASH – hybrid model superfluid nuclear model is 831.261 mb ($E_p = 15$ MeV) (Fig. 2). According to SRIM code, the required target thickness should be 49 μm. The $^{76}$Se(p,n)$^{76}$Br reaction, in the chosen range, led to form the $^{76}$Br im-

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**Fig. 2.** Excitation function of $^{76}$Se(p,n)$^{76}$Br reaction (a) by TALYS-1.2 code (b) via experimental data [11], level density options of ALICE/ASH 0.1 and TALYS-1.2 codes.
purity. The separation of isotope impurities is possible by chemical methods, so this reaction is carrier free for $^{76}$Br production. Over the last 15 years, this process has become the most frequently used reaction for the production of $^{76}$Br. The cross sections for this reaction have been measured by Levkovskij [30] and Hassan et al. [13]. The nuclear model calculations (performed as for $^{76}$Se(p,n)$^{76}$Br reaction) agree well with the measured values up to 25 MeV; but the data reported by Hassan et al. show strong inconsistencies; so nuclear model calculations can play an important role in removing the discrepancies.

Excitation function of $^{64}$Ni(p,n)$^{64}$Cu reaction

According to ALICE/ASH and TALYS-1.2 codes, beneficial energy range of the projectile particle (proton) to produce $^{64}$Cu from $^{64}$Ni target is 13 to 8 MeV among which maximum cross section by ALICE/ASH – hybrid model Kataria-Ramamurty formula with shell corrections 812.13 mb ($E_p = 11$ MeV). Carrier-free $^{64}$Cu production can be obtained using proton energy of less than 11 MeV. According to calculations from SRIM code the required target thickness should be 101 μm. The results of nuclear model calculations by the two codes with the measurement by Szelecsényi et al. [52], Avila-Rodriguez et al. [2] and Rebeles et al. [40] are shown in Fig. 3. The results of ALICE/ASH and TALYS-1.2 codes are in good agreement with the measured data from Rebeles et al., but experimental data by Szelecsényi et al. and Avila-Rodriguez et al. are lesser than two codes of Rebeles et al.

Excitation function of $^{66}$Zn(p,n)$^{66}$Ga reaction

According to ALICE/ASH and TALYS-1.2 codes, beneficial energy range of the projectile particle (proton) to produce $^{66}$Ga from $^{66}$Zn target is 15 to 9 MeV among
which maximum cross section by ALICE/ASH-Hybrid Model-Superfluid Nuclear Model is 683.3 mb \((E_p = 13\ \text{MeV})\). In using \(^{66}\text{Zn}(p,n)\)^{66}\text{Ga}\) reaction to produce \(^{66}\text{Ga}\), the best range of the incident energy was assumed to be 13 to 9 MeV. For this reaction, five cross-section measurements exist in the literature (Fig. 4). Experimental data that have been studied by Szelecsényi et al. [53], Hille et al. [16], Levkovskij [30], Little et al. [31] and Hermann et al. [15]. Experimental data by Little et al. and Hermann et al. are lesser than those of Szelecsényi et al., Hille et al. and Levkovskij. ALICE/ASH code agree well with the measured data from Szelecsényi et al., Hille et al. and Levkovskij up to 25 MeV. Also, the results of TALYS-1.2 code are in good agreement with the measured data by Hermann et al.

Excitation function of \(^{43}\text{Ca}(p,n)\)^{43}\text{Sc}\) reaction

Using \(^{43}\text{Ca}(p,n)\)^{43}\text{Sc}\) reaction to produce \(^{43}\text{Sc}\), the best range of incident energy was assumed 6 to 13 MeV, the maximum cross section by TALYS-1.2 code being 394.76 mb \((E_p = 9\ \text{MeV})\) (Fig. 5). Also maximum cross section of level density options of ALICE/ASH by Fermi gas level density \((a = A/18)\) is 348.93 mb \((E_p = 7.5\ \text{MeV})\). According to SRIM code, the required target thickness should be 220 \(\mu\text{m}\). The separation of isotope impurities is possible by chemical methods, so this reaction is carrier free for \(^{43}\text{Sc}\) production. This reaction was investigated only by de Waal et al. [9] and Levkovskij [30]. Experimental data and options of ALICE/ASH code are lesser than TALYS-1.2 code. De-

**Fig. 4.** Excitation function of \(^{66}\text{Zn}(p,n)\)^{66}\text{Ga}\ reaction (a) by TALYS-1.2 code (b) via experimental data [11], level density options of ALICE/ASH 0.1 and TALYS-1.2 codes.
spite the existing inconsistencies among the measured cross sections, the excitation function is described well by the model calculations. The evaluated cross-section curve is also shown in Fig. 5.

Conclusions

PET is the most sensitive method to image trace amounts of molecules in vivo. The production of $^{86}$Y, $^{43}$Sc, $^{64}$Cu, $^{66}$Ga and $^{76}$Br can be achieved by $^{86}$Sr(p,n)$^{86}$Y, $^{43}$Ca(p,n)$^{43}$Sc, $^{66}$Zn(p,n)$^{66}$Ga, $^{64}$Ni(p,n)$^{64}$Cu and $^{76}$Se(p,n)$^{76}$Br reactions ideal reaction for low energy cyclotrons. Moreover, they are non-carrier added production feasibility using proton energy considered as a brilliant advantage.

References

Nuclear data for the cyclotron production of $^{66}$Ga, $^{86}$Y, $^{76}$Br, $^{64}$Cu and $^{43}$Sc in PET imaging

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