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Finding optimal of the Egyptian second nuclear reactor core patterns using genetic algorithm

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Abstract. The second Egyptian research reactor ETRR-2 went critical on the 27th of November 1997. The National Center of Nuclear Safety and Radiation Control (NCNSRC) has the responsibility of the evaluation and the assessment of the safety of this reactor. Fuel management reloads for Egypt's second research reactor have been carried out according to the fuel management scheme suggested by the reactor designer (INVAP). The start up core consists of three different fuel types, while the equilibrium core has only one fuel type called standard fuel. The fuel management scheme consists in considering the core as being partitioned into eight zones. Each zone will correspond to a chain of fuel movements. In each fuel cycle two of these chains will be involved, in which eight fuel elements will be moved, from them two spent fuel elements will be extracted and two fresh fuel elements will be inserted in the core. In this paper we solve a model as a one big nonlinear multi objective discrete optimization problem using genetic algorithm. Results are compared with INVAP values.

Key words: nuclear reactor • reloading fuel management • genetic algorithm

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Introduction

Egypt's second research reactor is a multi-purpose reactor of type materials test reactor (MTR) designed by the Argentinean company INVAP [9]. The reactor was constructed with the Egyptian anticipation by INVAP and went critical at the end of 1997. The reactor is 22 MW with uranium fuel enriched to 19.7%. It is cooled and moderated by light water and reflected by beryllium. The reactor core is situated inside an open pool surrounded by a chimney water injection system with forced upward cooling.

An important issue in the operation of the nuclear reactor is the safety design of a reload pattern for each cycle. Usually, a number of fuel bundles is discharged at the end of a cycle (EoC), and the same number of fresh bundles is inserted in the core, while all bundles are reshuffled to a configuration that is optimal with respect to some performance criterion. There are several strategies for reloading. We consider the situation in which at each reload, the number of discharged bundles is the same.

Having fixed the number of discharged bundles, it is possible to simply measure the remaining reactivity of the bundles at each EoC, and then find an optimal pattern using a subset of these bundles together with some fresh bundles. Another approach is to apply the same reload pattern every year. After repeating this for a number of years, the reactor will reach an equilibrium state, in which each cycle has the same characteristics.

One way to chose an optimization criteria is to search a loading pattern for which the cycle length will be a maximum before the reactor becomes subcritical, which means that the number of neutrons produced is less than the number of neutrons lost by absorption from the reactor, which stops the reactor operation.

In our paper, we state the problem as a multi objective problem and we solve it using the genetic algorithm to find an optimal solution for the reload fuel pattern for the Egyptian second research reactor. This paper is organized as follows: in section 'Physical description' we give a general physical description background, in section 'Mathematical model' we develop the mathematical optimization model using those physical description, section 'The genetic algorithm' deals with a brief discussion of genetic algorithm also, we present our proposed algorithm for optimizing the Egyptian second nuclear reactor core patterns, in section 'Results and discussion' we discuss the results, the final conclusion being in section 'Conclusion'.

Physical description

In a nuclear reactor core, the desired energy is obtained by a controlled fission chain reaction. The fission of a nucleus into two smaller nuclei is induced by the capture of a free neutron producing a large amount of energy, together with some new free neutrons. The state of the reactor depends on the number of free neutrons in the core. The neutron flux, therefore, plays a major role in models describing the reactor processes.

A popular way to describe the neutron flux in the reactor core is to consider the production, absorption and transport of neutrons as a diffusion process. More detailed description can be found in [2, 3, 7].

Let X be the space of the core and the surrounding water, in a coordinate system that has yet to be specified, then for any position $x \in X$ we distinguish the following types of cross-sections:

- $\Omega_f^1(x)$, $\Omega_f^2(x)$ – fission cross-section of the fast and thermal group, the probability per unit path length that a neutron in the fast or thermal group will be absorbed by a nucleus to cause a fission reaction.
- $\Omega_a^1(x)$, $\Omega_a^2(x)$ – absorption cross-section of the fast and thermal group, the probability per unit path length that a neutron in the fast or thermal group will be absorbed by a nucleus,
- $\Omega_s(x)$ – down scattering cross-section from the fast to the thermal group, the probability per unit path length that a fast neutron is absorbed by a nucleus and immediately re-emitted at an energy level in the thermal group.

We will define the following parameters:

- $D^1(x), D^2(x)$ – the position dependent diffusion coefficients for the fast and thermal group,
- ν_1, ν_2 – the average number of fast neutrons, produced by fission reactions that are induced by neutrons in the fast and thermal group,
- υ_1, υ_2 – the average neutron speed in the fast and thermal group,
- $G_{i,j}$ – the probability that a neutron produced in the node j will be absorbed in the node i.

Given those parameters, we can now define the neutron in the core by a set of diffusion equations.

- $\phi^2(x,t)$, $\phi^2(x,t)$ – the neutron flux for the fast and thermal group.

So, we have to specify the problem dimensions. The following size constants are used: N is the number of nodes in the core; M is the number of discharged bundles at EoC; L is the number of age group "cycle time" in the core; $L = N/M$; T is the number of time steps; t is the time variables.

Also, we have to define the variables: $k^{\infty}_{i,t}$ is the average infinite multiplication factor of the bundle in node i; $\phi_{i,t}$ is the average fast neutron flux at node i; k_t^{eff} is the effective multiplication factor of the reactor core.

Then, we can write the set of time dependent diffusion equation for the fast and thermal group as [3]

$$(1)\quad \frac{1}{\nu_1}\frac{\partial\phi^1(x,t)}{\partial t} - \nabla_x.D^1(x,t)\nabla_x\phi^1(x,t) + \Omega_a^1(x)\phi^1(x,t) + \Omega_s(x)\phi^1(x,t) = \nu_1\Omega_f^1(x)\phi^1(x,t) + \nu_2\Omega_f^2(x)\phi^2(x,t)$$

$$(2)\quad \frac{1}{\nu_2}\frac{\partial\phi^2(x,t)}{\partial t} - \nabla_x.D^2(x,t)\nabla_x\phi^2(x,t) + \Omega_a^2(x)\phi^2(x,t) = \Omega_s(x)\phi^1(x,t)$$

These equations are stated in the general form:

$$\text{Neutron gain} = \text{rate of change} - \text{diffusion term} + \text{neutron loss}$$

when the neutron gain by production and neutron loss by absorption and diffusion are exactly equal, then the flux becomes independent of the time, and the reactor is said to be critical. So, under certain conditions, we may assume that $\phi(x,t)$ can be separated into $T(t)\,\phi(x)$, giving time dependent relations, that are not discussed here, and space dependent equations. These space dependent equations can only have solutions if we introduce an additional degree of freedom in the system.

This is obtained by introducing an eigenvalue λ to the system:

$$(3)\quad -\nabla_x.D^1(x)\nabla_x\varphi^1(x) + [\Omega_a^1(x) + \Omega_a^2]\varphi^1(x) = \lambda[\nu_1\Omega_f(x)\varphi^1(x) + \nu_2\Omega_f^2(x)\varphi^2(x)]$$

$$(4)\quad -\nabla_x.D^2(x)\nabla_x\phi^2(x,t) + \Omega_a^2(x)\phi^2(x,t) = \Omega_s(x)\phi^1(x,t)$$

So, the effective multiplication factor can be defined as follows:

$$(5)\qquad k^{\text{eff}} = \frac{\text{total neutron production rate}}{\text{total neutron loss rate}}$$

Then

$$\lambda = \frac{1}{k^{\text{eff}}}$$

Equation (4) can be rewritten since the thermal group diffusion is eliminated by neglecting the diffusion term, which can be possible since the neutron leakage in the thermal group is relatively small [3].

$$\phi^2(x) = \frac{\Omega_s(x)}{\Omega_a^2(x)}\phi^1(x) \tag{6}$$

which can be substituted in Eq. (3) to obtain the eigenvalue differential equation

$$-\nabla_x . D^1(x)\nabla_x \varphi^1(x) + \Omega_a^1(x)\varphi^1(x) + \Omega_s(x)\varphi^1(x) = \frac{1}{k^{\text{eff}}}[\nu_1\Omega_f(x) + \nu_2\Omega_f^2(x)]\varphi^1(x) \tag{7}$$

Some simplifications

We need to simplify (7) in order to derive our model. First, we will assume that the diffusion coefficient D^1, as well as some cross-sections are position independent. The fission cross-section Ω_f strongly depends on burn-up and is very position dependent. The absorption cross-section, especially for the thermal group, also depends on burn-up. However, in the 1/1/2 group approximation, the downscattering of neutrons is the most important mechanism of removing neutrons since we neglect thermal diffusion. This downscattering is almost independent of burn-up since it is mainly determined by the reactor coolant [10]. So, we can redefine:

$$D^1(x) \to D^1, \Omega_s(x) \to \Omega_s\Omega_a^1(x) \to \Omega_a^1, \Omega_a^2(x) \to \Omega_a^2$$

We now use $\Omega_f^1(x)$ and $\Omega_f^2(x)$ to define the new variable

$k^\infty(x)$ in the following way

$$k^\infty(x) = \left(\nu_1\Omega_f^1(x) + \nu_2\Omega_f^2(x)\frac{\Omega_s}{\Omega_a^2}\right) \Big/ (\Omega_a^1 + \Omega_s) \tag{8}$$

The variable k^∞, the infinite multiplication factor, is interpreted as the ratio of local neutron production and absorption. The eigenvalue differential Eq. (7) then transforms to

$$-L_f^2\nabla_x^2\phi(x) + \phi(x) = \frac{1}{k^{\text{eff}}}k^\infty(x)\phi(x) \tag{9}$$

with

$$L_f = \sqrt{D^1/(\Omega_a^1 + \Omega_s)}$$

known as the fast diffusion length.

Fuel burn-up

During reactor operation, the fission reactions cause burn-up of the fuel. Therefore, the fission cross-sections $\Omega_f^1(x)$ and $\Omega_f^2(x)$ decrease, also the absorption cross-sections also change. On the one hand, they decrease because of the decreasing atom density of fissionable nuclides. On the other hand, they increase because the fission products absorb neutrons, without causing a fission or scattering reaction. Moreover, most of the material is not fissionable and will not change in time [5]. We assume for the moment that we may neglect the changes in absorption cross-sections and only concentrate on changes in the fission cross-sections. So, we introduce time dependence on the following variables:

$$\Omega_f^1(x) \to \Omega_f^1(x,t), \Omega_f^2(x,t), k^{\text{eff}} \to k^{\text{eff}}(t), k^\infty(x) \to k^\infty(x,t), \phi(x) \to \phi(x,t)$$

where t ranges from the begin of a cycle (BoC) to the end of cycle (EoC).

$$\frac{\partial}{\partial t}\Omega_f^1(x,t) = -\sigma_a\Omega_f^1(x,t)\phi(x,t) \tag{10}$$

$$\frac{\partial}{\partial t}\Omega_f^2(x,t) = -\sigma_a\Omega_f^2(x,t)\phi(x,t) \tag{11}$$

where σ_a is the so-called microscopic cross-section.

From Eqs. (10) and (11) and by using Eq. (8) we can derive a description for the decay of $k^\infty(x,t)$:

$$\begin{aligned}\frac{\partial}{\partial t}k^\infty(x,t) &= \left(\nu_1\Omega_f^1(x,t) + \nu_2\Omega_f^2\frac{\Omega_s}{\Omega_a^1}\right)\Big/(\Omega_a^1 + \Omega_s) \\ &= -\sigma_a\left[\left(\nu_1\Omega_f^1(x,t) + \nu_2\Omega_f^2\frac{\Omega_s}{\Omega_a^1}\right)\Big/(\Omega_a^1 + \Omega_s)\right]\phi(x,t) \\ &= -\sigma_a k^\infty(x,t)\phi(x,t)\end{aligned} \tag{12}$$

Since k^∞ now is time dependent, the flux distribution also is time dependent, so, that we have to redefine the eigenvalue differential Eq. (9)

$$-L_f^2\nabla_x^2\phi(x,t) + \phi(x,t) = \frac{1}{k_t^{\text{eff}}}k^\infty(x,t)\phi(x,t) \tag{13}$$

$$\phi(x,t) = 0, \quad x \in \text{boundary of } x, \tag{14}$$

$$\frac{\partial\phi(x,t)}{\partial x} = 0, \quad x \text{ on symmetry edge} \tag{15}$$

$$\int_x \phi(x,t)k^\infty(x,t)dx = \frac{P_c}{\mu_f(\Omega_a^1 + \Omega_s)} \tag{16}$$

where, P_c is the required total power over the whole core, and μ_f is the amount of energy released per fission.

Then,

$$\varphi_{i,t} = \frac{1}{k_t^{\text{eff}}}\sum_{j=1}^{N} G_{i,j}k_{j,t}^\infty\varphi_{j,t}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T-1 \tag{17}$$

So, Eq. (16) becomes:

$$\sum_{i=1}^{N} k_{i,t}^\infty\varphi_{i,t} = \frac{P_c}{\mu_f(\Omega_a^1 + \Omega_s)}, \quad t = 1,\ldots,T \tag{18}$$

and the decay Eq. (12) is replaced by

$$k_{i,t+1}^\infty - k_{i,t}^\infty = -\sigma_a\Delta t k_{i,t}^\infty\varphi_{i,t}, \quad i = 1,\ldots,N, \quad t = 1,\ldots,T-1 \tag{19}$$

Mathematical model

In this section, we will develop a nodal core model that describes the reloading patterns and the corresponding evolution of the core from BoC to EoC. The evolution of the core during a cycle is computed in a fixed number of discrete time steps. At each EoC, the same fixed number of old fuel bundles is discharged, which are all of the same age. The objective of our optimization is to maximize the effective multiplication factor at EoC, while operational constraints are satisfied.

The necessary equations can be divided into four different types. The first class of equations describes the core evolution during the cycle, given the infinite multiplication factors at BoC. The second class specifies a loading pattern. The third class describes the reloading operation itself; here the equilibrium cycle property is specified. The fourth class are the operational constraints. We will introduce the four types in sequence.

$$R_{i,t} = \varphi_{i,t} \frac{\mu_f (\Omega_a^1 + \Omega_s)}{P_c}$$

and, we can introduce,

$$\alpha = \frac{\sigma_a}{\mu_f (\Omega_a^1 + \Omega_s)}$$

From Eqs. (17), (18) and (19) :

$$k_t^{\text{eff}} R_{i,t} = \sum_{j=1}^{N} G_{i,j} k_{j,t}^{\infty} R_{j,t}, \quad i = 1,...,N, \quad t = 1,...,T \tag{20}$$

$$k_{i,t+1}^{\infty} - k_{i,t}^{\infty} = -\alpha P_a \Delta t k_{i,t}^{\infty} \varphi_{i,t} R(i,t), \quad i = 1,...,N, \quad t = 1,...,T-1 \tag{21}$$

$$\sum_{j=1}^{N} k_{j,t}^{\infty} R_{j,t} = 1, \quad t = 1,...,T \tag{22}$$

Fuel management scheme

The start up core configuration set for the reactor operation as designed by INVAP is shown in Fig. 1. It consists of three types of fuel elements. In addition to the standard fuel elements, two other types with lower U-235 contents are used. These fuel types are as follows: 8 fuel elements of 148.2 gm U-235 per element; 14 fuel element of 209.0 gm U-235 per element; 7 fuel element of 404.7 gm U-235 per element.

The fuel management strategy suggested by the reactor designer for the reactor operation is based on considering the reactor core as being partitioned into eight zones. In every fuel cycle two zones are involved, in which two fresh fuel elements are inserted, one

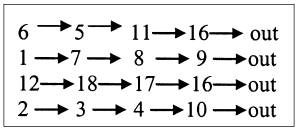

Fig. 1. The fuel movement in the periodic 4 fuel cycles.

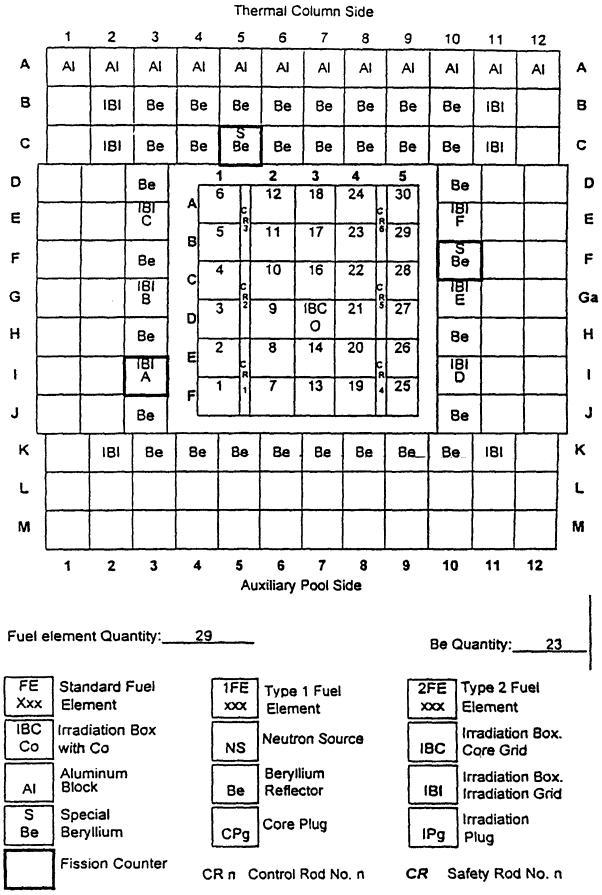

Fig. 2. First core configuration: FE – 404.7 gm U-235; 1FE – 148.2 gm U-235; 2FE – 209.0 gm U-235.

spent fuel element is removed and 4 fuel elements are shuffled in the core. By this fuel management scheme, the reactor is operated in periodic four fuel cycles. The movement in these 4 cycles is as follows [8].

From this representation, we can read that nodes 6, 5 and 11 always contain fresh bundles. During reloading, the bundle from node 16 is discharged, the one from 11 is moved to node 16, a fresh bundle is inserted at position 6, etc.

A fixed fuel cycle length was assumed, which was 18 full power days. Figure 2 illustrates the fuel movements in the periodic 4 fuel cycles.

Loading pattern specification

At each EoC, the same number of old fuel bundles is discharged, and all those removed bundles are of the same age. Here we take the Egyptian second research reactor as a case of study; we can describe the life of the bundles in the core during L cycles as a trajectory notation [12]. For example, suppose we have 16 nodes in the core, and at each EoC four bundles are discharged.

So, we can formulate the problem in optimization context by describing the trajectories in the following way $x_{i,l,m} = \{_0^1$ in which, $x_{i,l,m}$ equals 1, if node i will contain the bundle of age l from trajectories m equal 0, otherwise, where $i = 1, \ldots, N$; $l = 1, \ldots, L$; $m = 1, \ldots, M$, of course, we need to describe the necessary variables $x_{i,j,m}$ in a set of equations:

$$\sum_{l=1}^{L}\sum_{m=1}^{M} x_{i,j,m} = 1, \qquad i = 1,...,N$$

$$\sum_{i=1}^{N} x_{i,l,m} = 1, \quad l = 1,...,L, \quad m = 1,...,M$$

$$x_{i,l,m} \in \{0,1\}$$

Reloading operation

The reloading equations consists of two terms; in the first term we use the variables $x_{i,l,m}$ to determine whether node i contains a fresh bundle after reloading, and assign to $k^{\infty}_{i,1}$ the value k^{fresh} if this is the case. If not, then the second term determines from $x_{i,l,m}$ whose bundle (l, m) is in node i. The total reloading equation is given by

$$(23) \qquad k^{\infty}_{i,1} = \sum_{m=1}^{M} x_{i,1,m} k^{\text{fresh}} + \sum_{l=2}^{L}\sum_{m=1}^{M} x_{i,l,m} \sum_{j=1}^{N} x_{j,l-1,m} k^{\infty}_{j,T} \qquad i = 1,...,N$$

We can simplify the reloading equation in the form

$$(24) \sum_{i=1}^{N} x_{i,l,m} k^{\infty}_{i,1} = \begin{cases} \sum_{j=1}^{N} x_{j,l-1,m} k^{\infty}_{j,T}, & l = 2,...,m, \quad m = 1,...,M \\ k & l = 1, m = 1,...M \end{cases}$$

Not all loading patterns are allowed. For safety reason, it is required that the maximal power in one bundle is not too large. This peaking constraint is applied for every node at any time, and it is stated as follows:

$$(25) \qquad k^{\infty}_{i,t} R_{i,t} \le \frac{f^{\lim}}{N}, \quad i = 1,...,N, \quad t = 1,...,T$$

where $f^{\lim}$ is a value greater than 1.

Suppose a bundle from a non diagonal position has to be reloaded into a diagonal position. Because this diagonal position belongs to two octants, so, we can require that a bundle is assigned to two diagonal positions simultaneously, with a half the volume in one position, and a half the volume in the order position. When in the next cycle these two half bundles are placed in one node, the composed bundle has at BoC average properties of two diagonal boundless at EoC. This is modeled by introducing a volume vector $V = V_1,\ldots,V_N$, where V_i is ½ if i is the diagonal node, and V_i is 1 for all other nodes. Now, the number of different ages in the core L times the number of fresh bundles M is no longer equal to N, the number of nodes in the octant core is

$$LM = \sum_{i=1}^{N} V_i$$

The restriction that each bundle is in exactly one node, now is replaced by the restriction that each bundle is in one or two nodes with total volume 1

$$\sum_{i=1}^{N} V_i x_{i,l,m} = 1$$

After reloading, a bundle is given the average properties of its predecessors:

$$k^{\infty}_{i,1} = \sum_{m=1}^{M} x_{i,1,m} k^{\text{fresh}} + \sum_{l=2}^{L}\sum_{m=1}^{M} x_{i,l,m} \sum_{j=1}^{N} V_j x_{j,l-1,m} k^{\infty}_{j,T}, \qquad i = 1,...N$$

The power normalization has to be performed on the octant core:

$$\sum_{i=1}^{N} V_i k^{\infty}_{i,t} R_{i,t} = 1$$

So, the objective of the optimization is to maximize the effective multiplication factor at EoC, we can formulate it as follow

$$\max k^{\text{eff}}_{T}.$$

Now, we can list a complete model of the octant core:

$$\max_{x,k^{\infty},R,k^{\text{eff}}} k^{\text{eff}}_{T}$$

Subject to:

$$\sum_i V_i x_{i,l,m} = 1, \qquad l = 1,...L, \quad m = 1,...,M$$

$$\sum_l \sum_m x_{i,l,m} = 1, \qquad i = 1,...,N$$

$$k^{\infty}_{i,1} = \sum_{l>1} \sum_m x_{i,l,m} \sum_j V_i x_{j,l-1,m} k^{\infty}_{j,T} + (\sum_m x_{i,1,m}) k^{\text{fresh}}, \qquad i = 1,...,N$$

$$k^{\text{eff}}_t R_{i,t} = \sum_j G_{i,j} k^{\infty}_{j,t} R_{j,t}, \qquad i = 1,...,N, \quad t = 1,...,T$$

$$k^{\infty}_{i,t+1} = k^{\infty}_{i,t} - (\alpha P_c \Delta t) k^{\infty}_{i,t} R_{i,t}, \quad i = 1,...,N, \quad t = 1,...,T-1$$

$$\sum_i V_i k^{\infty}_{i,t} R_{i,t} = 1, \qquad t = 1,...,T$$

$$k^{\infty}_{i,t} R_{i,t} \le f^{\lim} \Big/ \sum_j V_j, \qquad i = 1,...,N, \quad t = 1,...,T$$

where

$$k^{\text{eff}}_t \ge 0, \qquad t = 1,...,T$$

$$k^{\infty}_{i,t} \ge 0, \qquad i = 1,...,N, \quad t = 1,...,T$$

$$R_{i,t} \ge 0, \qquad i = 1,...,N, \quad t = 1,...,T$$

$$x_{i,l,m} \in \{0,1\} \quad i = 1,...,N, \quad l = 1,...,L, \quad m = 1,...,M$$

The genetic algorithm

Genetic algorithms (GA's) were introduced by Holland in the 1970s [6] not to solve a particular problem, but to investigate the effects of natural adaptation in stochastic search algorithms. GA uses three main types of rules at each step to create the next generation from the current population. Selection rules select the individuals, called parents that contribute to the population at the next generation. Crossover rules combine two parents to form children for the next generation. Mutation rules apply random changes in individual parents to form children computation, select the next population by computations that involve random choices [11]. The simple genetic algorithm follows the structure depicted in Fig. 3. Each of these operations will be described in the following subsections [4].

Selection plays a central role in genetic algorithms determining how individuals compete for gene survival. Selection weeds out the bad solutions and keeps the good ones. The goal of GA is essential in finding a set of parameters that maximize or minimize the output of a function or fitness. An individual with the best fitness is then selected to be a part of the next generation.

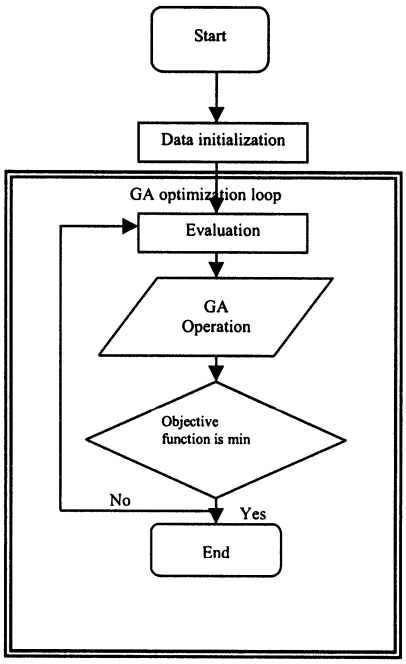

Fig. 3. Simple genetic algorithm.

Selection in GA's is usually done on the whole original population and usually is repeated for all individuals in the population [13].

The simple genetic algorithm can be mathematically described as:

– For each chromosome $S_i, i = 1,2,\ldots pop\_size$ compute the fitness value

$$eval(S_i) = f(S_i) \tag{26}$$

– Compute the total fitness of the population

$$F = \sum_{i=1}^{pop\_size} eval(S_i) \tag{27}$$

– Compute the probability of a selection p_i for each chromosome S_i

$$p_i = eval(S_i)/F \tag{28}$$

– Compute the cumulative probability of a selection q_i for each chromosome S_i

$$q_i = \sum_{j=1}^{i} p_j \tag{29}$$

– Generate a random number $r \in [0,1]$
– If $r \prec q_1$ then select the first chromosome S_1, otherwise select the i-th chromosome S_i, $i = 1,2,\ldots$ pop_size such that $q_{i-1} \prec r \prec q_1$.

Recombination of individuals is done to investigate the performance of new individuals that resemble the exiting ones. Recombination is often done by crossover [1]. In the crossover phase, all of the chromosomes (except for the elite chromosome) are paired up, and with a crossover probability p_c, they are crossed over. The crossover is accomplished by randomly choosing a site along the length of the chromosome, and exchanging the genes of the two chromosomes for each gene past this crossover site [14]. The crossover operation proceeds in the following manner:

– for each chromosome S_i in the population,
– generate a random number $r \in [0,1]$,
– if $r \prec p_c$ then select the given chromosome for crossover,
– compute select chromosomes randomly,
– for each pair of coupled chromosomes, generate a random integer number $pos \in [1,\ldots m-1]$, m is the number of bits in each chromosome, the number pos indicates the position of crossing point the following tow chromosomes are crossed over as follows:

$$(b_1 b_2 \ldots b_{pos} b_{pos+1} b_m) \text{ and } (c_1 c_2 \ldots c_{pos} c_{pos+1} c_m) \tag{30}$$

Are replaced by a pair of their offspring

$$(b_1 b_2 \ldots b_{pos} c_{pos+1} c_m) \text{ and } (c_1 c_2 \ldots c_{pos} b_{pos+1} b_m) \tag{31}$$

Mutation. After crossover, for each of the genes of the chromosomes (except for the elite chromosome), the gene will be mutated to any one of the codes with a mutation probability p_m. With the crossover and mutations completed, the chromosomes are once again evaluated for another round of selection and reproduction. Even if most of the search is being performed by recombination, mutation can be vital to provide the diversity which recombination needs. The mutation operation proceeds in the following manner:

For each chromosome in the population applies:

– generate a random number $r \in [0,1]$
– if $r \prec p_m$ mute this bit by a change of its value from 0 to 1 or *vice versa*.

Proposed algorithm

The solution procedure of the proposed algorithm is summarized in the following steps:

1. Set $k = 0$.
2. Initialize the two populations $P_s(k)$ and $P_r(k)$ randomly. Where $E(k)$ is init. by zero.
3. If one reference point $r \in M$ is reached, then go to 7.
4. Double the number of trials to obtain a reference point $r \in M$. If it is reached, then go to 7.
5. Increase the precision parameter of the algorithm, if it is reached go to 7.
6. Read the solution of the dual problem as a reference point. Go to 7.
7. Evaluate the population $P_r(k)$ by using the objective function, and sort $P_r(k)$.
8. Update the elitist point $E(k)$ by the reference point has the best fitness.
9. Check feasibility, if the search point S of the population $P_s(k)$ are feasible, evaluate $P_s(k)$ using the objective function, go to 12.
10. Create a random point $z \in M$ from a segment line between s and $E(k)$ as: $z = \delta_s + (1 - \delta) E(k)$, where $\delta \in [0,1]$ is a random number.
11. Evaluate the point z, if the fitness of z is better than that of $E(k)$, then replace $E(k)$ and the most fit reference point r by z. Also, replace s by z with some probability of replacement.

12. If the stopping rule is satisfied, then go to step 18; elor set $k = k + 1$.
13. Select the population $P_s(k)$ from $P_s(k-1)$ using a ranking selection method.
14. Recombine the new population $P_s(k)$ by using genetic operators.
15. If the number of $k/n = 0$, then go to 6, elor go to 9.
16. Recombine the new population $P_r(k)$ by using genetic operators.
17. Select the population $P_r(k)$ from $P_r(k-1)$ using a ranking selection method, go to 7.
18. Stop.

Results and discussion

First of all, we must setup of problem we can have:

– First, we have to supply starting values that have some physical meaning. It is known that fresh bundles in the middle lead to a large value of the power peak. On the other hand, the bundles on the boundaries of the core should be sufficiently burned, since this causes a loss of neutrons in the surrounding water. It is known that good patterns, therefore, have a sort of a ring structure, in which old bundles are put on the boundaries, and fresh bundles somewhere in the middle. Although we want to use a starting point that is based on such known good patterns, so, a fractional starting pattern is constructed, in which it is specified how many percent of each bundle is initially in each node, see Fig. 4.

– Decreasing k^{eff}. During the cycle, the value of the eigenvalue k^{eff} decreases. Since the problem is very sensitive to the value of k^{eff}, we added the set of constraint.

$$k_t^{\text{eff}} \geq k_{t+1}^{\text{eff}}, \quad t = 1,...,T-1 \tag{32}$$

– Relaxation of the power peaking constraint. For a fixed pattern, physical equations describe the evolution of the core, and there always exists a feasible solution with respect to those equations. The power peaking restricts the number of feasible reloading patterns. Motivated by this observation, we initially relax the power peaking constraint with some perturbation variable ε.

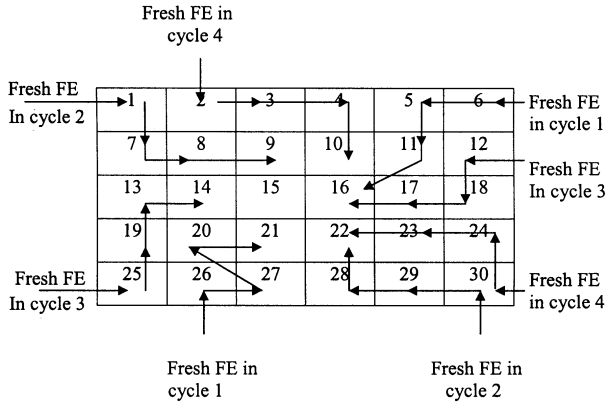

Fig. 4. First shuffling scheme in periodic 4 cycle.

$$k_{i,t}^{\infty}\varphi_{i,t} \leq \frac{f^{\lim}}{\sum_{j=1}^{N} V_j} \tag{33}$$

We implemented

$$k_{i,t}^{\infty}\varphi_{i,t} \leq (1+\varepsilon)\frac{f^{\lim}}{\sum_{j=1}^{N} V_j} \tag{34}$$

The variable ε is non-negative, it is penalized in the objective function with the penalty parameter ω, which is set to 1. So, the objective now becomes

$$\max_{x,k^{\infty},R,k^{\text{eff}}} k_T^{\text{eff}} - \omega\varepsilon \tag{35}$$

we set the genetic parameters used as follows:

$N_{ger} = 100$, $N_{pop} = 100$, Mutation Pr $ob. = 0.05$. The graphic results in Fig. 5 shows the results obtained through a genetic algorithm procedure for maximizing our objective functions of mean unavailability, it shows the best fitness function for all of them. We report the values of the objective functions as shown in Table 1, which listed 30 test cases. The restart from the best infeasible neighbor turned out to be helpful in almost half of the case. On the other hand, it shows that in a few cases it caused a longer search without improving the objective function.

Table 1. Results for 30 different test problems

Prob.	Objective values	Times
Small		
1	1.01114	74.5
2	1.04848	73.1
3	1.02051	80.8
4	1.04275	94.4
5	1.03975	135.8
6	1.03407	92.7
7	1.02753	150.6
8	1.02793	79.6
9	1.02426	68.1
10	1.02475	136.5
Medium		
1	1.03537	385.9
2	1.03510	3348.2
3	1.03492	3506.1
4	1.03429	4125.9
5	1.03380	3467.4
6	1.03558	3855.6
7	1.03349	3002.7
8	1.03548	4630.7
9	1.03402	4084.6
10	1.03329	3562.1
Large		
1	1.04756	81 468.1
2	1.04738	61 920.2
3	1.04724	80 259.1
4	1.04658	39 161.3
5	1.04646	60 335.1
6	1.04757	53 541.8
7	1.04675	64 471.6
8	1.04739	38 945.7
9	1.04665	52 791.8
10	1.04623	50 714.4

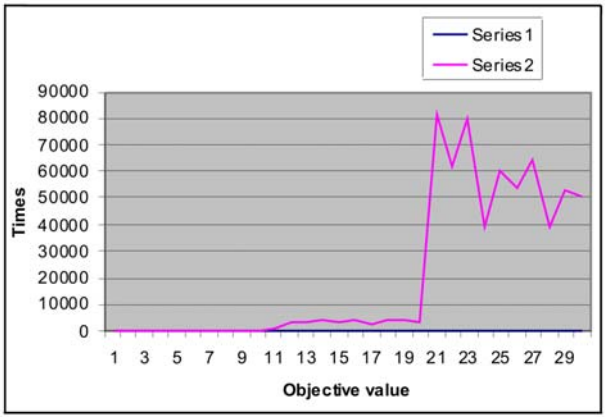

Fig. 5. The results for genetic algorithm procedure for maximizing our objective functions.

These results certainly constitute a more informative set which the designer can handle for a more informed decision. The main features of the proposed algorithm could be summarized as follows: the proposed algorithm uses a elitist strategy to keep the best solution discovered up to the current generation. Also, the use of this strategy guarantees that the best fitness of the population never decreases from one generation to the next, the elitist strategy produces a faster convergence of the algorithm for optimal solution. Our proposed algorithm utilizes the idea of the weak duality theorem, such that both primal and dual solutions of the nonlinear programming problem are generated simultaneously, to determine the interval in which the optimal solution is located at any generated number of iterations. Also, to verify the convergence for the optimal solution regardless of the computational time of the algorithm. This proposed algorithm deals with constraints in a direct way instead of using penalty functions for handling constraints. Our final proposed algorithm uses three chromosomes to represent the solution vectors; they are modeled as floating point numbers representation.

Conclusion

Genetic algorithm optimization is a powerful tool for optimization of a given objective function, subject to a number of constraints. A simplified mathematical model for describing reloading pattern design in the Egyptian second nuclear reactor has been formulated here as a nonlinear optimization problem. The objective of the optimization is to maximize the effective multiplication factor at EoC. One of the strong advantage of nonlinear multi objective optimization using genetic algorithm is its capability to handle other, by nature continuous optimization problems. It is interesting to study the behavior of a one stage optimization procedure optimizing both the reloading pattern and a burnable over the fresh bundles. Also it is so important to mention that implicit formulations of the problem are allowed, since inside such an optimization procedure a Newton like method is used to find a feasible solution.

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