Solution of diffusion-advection equation of radon transport in many-layered geological media

Valentina S. Yakovleva, Roman I. Parovik

Abstract. Radon transport modeling in geological media is an important tool for solving problems and tasks of radioecology and geophysics. Comparison of radon field time series obtained by numerical and experimental methods is one of the most common and widely applicable ways to analyze the influence of state and variability of meteorological, electrical and actinometric parameters of atmosphere, cosmic weather factors, variations of deflected mode of geological medium on the level and variations of radon field. The solutions of stationary and non-stationary diffusion-advection equations of radon transport in many-layered geological media by numerical methods, notably by integro-interpolation method (balance method) are presented. The peculiarity of radon transport in many-layered media is taken into account in the developed numerical model. This peculiarity is connected with the transport equation coefficients which can change very rapidly at the border of two adjacent layers, i.e. they can be discontinuous at the borders of each layer that can be caused by parameters of soils greatly differing in value (density, porosity, radium content, diffusion and emanation coefficients). The present work is provided with an example of application of the developed numerical model for solving a practical problem on assessment of influence of deep seated uranium-containing rock on the value of radon volumetric activity at the depth of ≤ 1 m. The article considers non-stationary numerical model calculations showing at what time moments the distribution curves of radon volumetric activity coincide with stationary regime of radon transport in geological media. The validity of the developed numerical solution has been confirmed by these calculations.

Key words: radon • diffusion • advection • transport • modeling • soil

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Received: 29 June 2009
Accepted: 30 December 2009

Introduction

Radon transport modeling in geological media is widely applicable to solve practical problems and to explore the regularities of radon behavior [1–4, 6, 9, 10]. Based upon mathematical model the assessment of transport parameters is performed and predictions applicable in different spheres are made, for example: in building to amend the Code of Practice; in geological exploration to explore uranium ore deposits; in radioecology to assess radon danger for areas and buildings; in geophysics to study lithosphere-atmosphere bonds.

Radon transport modeling in geological media, which are described as similar to real ones, is a difficult issue, because geological medium is heterogeneous, layered with significantly different geophysical and geochemical characteristics for each layer. In this case a model of radon transport in layered medium with non-constant coefficients is required. Model coefficients can be the functions of spatial and time coordinates, and moreover, they can change very rapidly at a border of two adjacent layers, i.e. they can be discontinuous at the borders of each layer that can be caused by greatly differing parameters of soils (density, porosity, radium content, diffusion and emanation coefficients).

In the explicit form there is no way to obtain analytical solution for such a model, therefore, in this case numerical methods are applicable.

The present work deals with obtaining numerical solution for stationary and non-stationary diffusion-advection equation for radon transport in many-layered geological media.

Problem of stationary radon transport in many-layered medium with discontinuous coefficients

Let us first consider the problem of stationary radon transport in many-layered medium with discontinuous coefficients, and then apply the modeling results to the case of non-stationary transport. As coefficients of transport equation can be discontinuous, then at the border of each layer the conditions of the so-called ideal contact (continuity) are specified, notably the equality of fluxes and volumetric activities of radon.

Formulation of the problem

Stationary radon transport in many-layered porous medium occurs by means of diffusion-advection mechanisms [5, 6]. It is required to assign distribution of radon volumetric activity along the depth of geological medium. According to such assignment the radon transport equation in medium for N-layered structure will have the following form (z-axis is directed downward from the ground surface, $z \geq 0$).

$$\begin{aligned}
&\frac{d}{dz}\left(D_1(z)\frac{dA_1(z)}{dz}\right)+\upsilon_1(z)\frac{dA_1(z)}{dz}-\lambda(A_1(z)-A_{1,\infty})=0\\
&\frac{d}{dz}\left(D_n(z)\frac{dA_n(z)}{dz}\right)+\upsilon_n(z)\frac{dA_n(z)}{dz}-\lambda(A_n(z)-A_{n,\infty})=0\\
&\frac{d}{dz}\left(D_{n+1}(z)\frac{dA_{n+1}(z)}{dz}\right)+\upsilon_{n+1}(z)\frac{dA_{n+1}(z)}{dz}-\lambda(A_{n+1}(z)-A_{n+1,\infty})=0\\
&\frac{d}{dz}\left(D_N(z)\frac{dA_N(z)}{dz}\right)+\upsilon_N(z)\frac{dA_N(z)}{dz}-\lambda(A_N(z)-A_{N,\infty})=0
\end{aligned} \quad (1)$$

Conditions at the external borders of the whole many-layered structure have the following form

$$A_1(0)=0, \lim_{z\to\infty} A_N(z)=A_{N,\infty} \quad (2)$$

Here $A_n(z)$ is the radon activity per unit volume of porous medium (Bq·m^{-3}); subscript $n = 1,\ldots,N$ indicates the layer number; N is the quantity of layers; l_n is the thickness of n-layer; υ_n is the advective radon transport velocity in n-layer (m·s^{-1}); D_n is an effective diffusion coefficient in n-layer (m^2·s^{-1}); λ is the radon decay constant (s^{-1}); $A_{n,\infty}$ is the radon volumetric activity being in radioactive equilibrium with ^{226}Ra in n-layer, equal to $A_{n,\infty} = K_{n,em}A_{n,\mathrm{Ra}}\rho_{n,s}(1-\eta_n)$, where $K_{n,em}$ is the emanation coefficient for layer n in relative units; $A_{n,\mathrm{Ra}}$ is the specific activity of ^{226}Ra for layer n (Bq·kg^{-1}); $\rho_{n,s}$ is the solid soil particle density for n-layer (kg·m^{-3}); η_n is the soil porosity for n-layer in relative units.

Method of solution

Let us consider a calculation region of the problem (Eqs. (1), (2)). Let the thickness of geological medium l be a rather large value. Then, it can be expressed as the sum of appropriate layers: $l = l_1 + l_2 + \ldots + l_{n-1} + l_n + l_{n+1} + \ldots + l_N$. Let us divide the layer thickness into K_1 points, then $l_1 = i_1h_1$, where $i_1 = 0,1,2,\ldots,K_1 - 1$, h_1 – a grid pitch for layer l_1. Similarly for layer l_2 we have $l_2 = i_2h_2$, where $i_2 = K_1, K_1 + 1,\ldots, K_2 - 1$. Carrying on the process for n-layer we will obtain: $l_n = i_nh_n$, where $i_n = K_{n-1}, K_{n-1} + 1,\ldots, K_n - 1$. Finally, for the last layer we will have $l_N = i_Nh_N$, where $i_N = K_{N-1}, K_{N-1} + 1,\ldots,K_N - 1, K_N$. For simplicity, we can assume that $h = h_1 = h_2 = \ldots = h_n = \ldots = h_N$, i.e. we will consider the uniform grid by spatial coordinate. Then, for the whole soil thickness the correlation will be performed: $l = ih$, where $i = 0,1,2,\ldots, K_1 - 1, K_1, K_1 + 1,\ldots,K_2 - 1, K_2, K_2 + 1,\ldots,K_n - 1, K_n, K_n + 1,\ldots, K_N - 1, K_N$. The region under consideration is represented schematically in Fig. 1.

For problem (Eq. (1), (2)) solution, the integro-interpolated method is applied. According to this method there occurred a transition from a system of differential equations (1) to algebraic system expressed in a difference form. Such transition occurs via some integral relation (balance equation) expressing the conservation law for small volume [8]. The integrals and derivatives contained in balance equation should be substituted by approximate difference expressions. Thus, to solve the problem of radon transport in layered media we apply integro-interpolated method (balance method) of constructing conservative difference scheme.

For Eq. (1) in n-layer let us write down the balance equation for intercept $z_{i_n-1/2} \leq z \leq z_{i_n+1/2}$ here $z_{i_n-1/2} = h(i_n - ½)$, h – pitch of difference scheme [8].

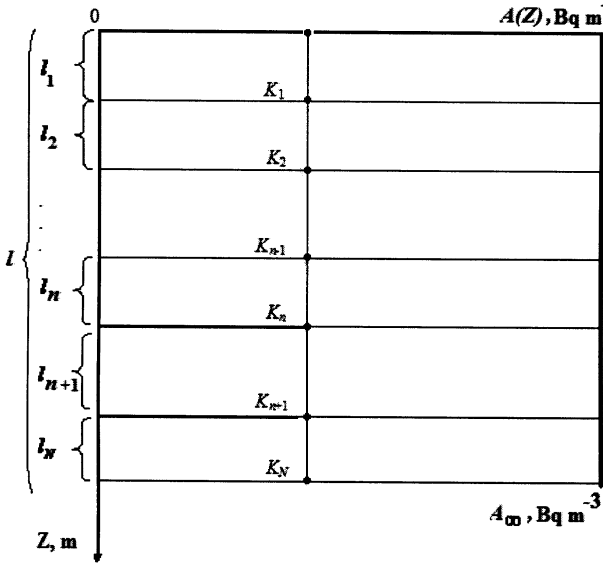

Fig. 1. The region under consideration for the problem (Eqs. (1), (2)).

$$\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} \frac{d}{dz}\left(D_n(z)\frac{dA_n(z)}{dz}\right)dz + \int_{z_{i_n-1/2}}^{z_{i_n+1/2}} \upsilon_n(z)\frac{dA_n(z)}{dz}\,dz - \lambda\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} A_n(z)\,dz = -\lambda\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} A_{n,\infty}(z)\,dz \tag{3}$$

Balance Eq. (3) reflects the conservation law for intercept $z_{i_n-1/2} \le z \le z_{i_n+1/2}$. In order to obtain difference equation from balance Eq. (3), it is necessary to use the approximations of grid functions. We seek the function of solution at integer nodes ($A(z)$, $z = z_{i_n}$), and diffusion and advection fluxes – at half-integer nodes. Let us imagine the first integral as diffusion fluxes difference $[(q(z) = -D_n(z)\cdot(dA_n(z)/dz),\ z = z_{i_n+1/2}]$ at half-integer node points and write down their approximation according to work [8].

$$q_{n,i_n+1/2} - q_{n,i_n-1/2} = \frac{1}{h}\begin{pmatrix} D_{n,i_n+1/2}A_{n,i_n+1} \\ -(D_{n,i_n+1/2}+D_{n,i_n-1/2})A_{n,i_n} \\ +D_{n,i_n-1/2}A_{n,i_n-1}\end{pmatrix} \tag{4}$$

Let us further perform approximation of the second integral, which reflects advective radon flux, by quadrature trapezium rule

$$\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} \upsilon_n(z)\frac{dA_n(z)}{dz}dz \approx \frac{\upsilon_{n,i_n-1/2}}{2}(A_{n,i_n} - A_{n,i_n-1}) + \frac{\upsilon_{n,i_n+1/2}}{2}(A_{n,i_n+1} - A_{n,i_n}) \tag{5}$$

Let us perform approximation of other equation terms (radon decay and formation) by functions whose values are sought at integer nodes of the grid

$$\lambda\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} A(z)dz \approx \lambda h A_{i_n}, \qquad \lambda\int_{z_{i_n-1/2}}^{z_{i_n+1/2}} A_{n,\infty}(z)dz \approx \lambda h A_{n,\infty i_n} \tag{6}$$

Let us substitute expressions (4), (5) and (6) into Eq. (3), and this will lead us to the system of algebraic equations of the following form

$$\begin{aligned} &-\alpha_{n,i_n}A_{n,i_n-1} + \gamma_{n,i_n}A_{n,i_n} - \beta_{n,i_n}A_{n,i_n+1} = F_{n,i_n} \\ &\alpha_{n,i_n} = \frac{D_{n,i_n-1/2}}{h^2} - \frac{\upsilon_{n,i_n-1/2}}{2h}, \\ &\gamma_{n,i_n} = \frac{D_{n,i_n+1/2}+D_{n,i_n-1/2}}{h^2} + \frac{\upsilon_{n,i_n+1/2}-\upsilon_{n,i_n-1/2}}{2h} + \lambda \\ &\beta_{n,i_n} = \frac{D_{n,i_n+1/2}}{h^2} + \frac{\upsilon_{n,i_n+1/2}}{2h},\ F_{n,i_n} = \lambda A_{n,\infty i_n}\end{aligned} \tag{7}$$

We solve system (7) for n-layer by double sweep method

$$\begin{aligned} &A_{n,i_n} = p_{n,i_n+1}A_{n,i_n+1} + q_{n,i_n+1}, \\ &i_n = K_{n-1}, K_{n-1}+1, K_{n-1}+2..., K_n-2 \\ &p_{n,i_n+1} = \frac{\beta_{n,i_n}}{\gamma_{n,i_n} - p_{n,i_n}\alpha_{n,i_n}}, \\ &q_{n,i_n+1} = \frac{\beta_{n,i_n}q_{n,i_n} + F_{n,i_n}}{\gamma_{n,i_n} - p_{n,i_n}\alpha_{n,i_n}}\end{aligned} \tag{8}$$

Similarly, applying algorithm for $n + 1$ layer, we will obtain the solution

$$\begin{aligned} &A_{n+1,i_{n+1}} = p_{n+1,i_{n+1}+1}A_{n,i_{n+1}+1} + q_{n+1,i_{n+1}+1} \\ &i_{n+1} = K_n+2, K_n+3, ..., K_{n+1}-1, K_{n+1}, \\ &p_{n+1,i_{n+1}+1} = \frac{\beta_{n+1,i_{n+1}}}{\gamma_{n+1,i_{n+1}} - p_{n+1,i_{n+1}}\alpha_{n+1,i_{n+1}}} \\ &q_{n+1,i_n+1} = \frac{\beta_{n+1,i_n}q_{n+1,i_{n+1}} + F_{n+1,i_{n+1}}}{\gamma_{n+1,i_{n+1}} - p_{n+1,i_{n+1}}\alpha_{n+1,i_{n+1}}}\end{aligned} \tag{9}$$

Finally, the solution of problem (1, 2) can be written as

$$\begin{aligned} &A_{1,0} = 0,\ A_{1,i} = p_{1,i+1}A_{1,i+1} + q_{1,i+1}, \\ &\quad A_{1,K_1} = p_{1,K_1+1}A_{1,K_1+1} + q_{1,K_1+1}, \\ &A_{2,i} = p_{2,i+1}A_{2,i+1} + q_{2,i+1}, A_{n,i} = p_{n,i+1}A_{n,i+1} + q_{n,i+1}, \\ &A_{n,K_n} = p_{n,K_n+1}A_{n,K_n+1} + q_{n,K_n+1}, \\ &A_{N-2,i} = p_{N-2,i+1}A_{N-2,i+1} + q_{N-2,i+1}, \\ &A_{N-2,K_{N-2}} = p_{N-2,K_{N-2}+1}A_{N-2,K_{N-2}+1} + q_{N-2,K_{N-2}+1}, \\ &\quad A_{N-1,i} = p_{N-1,i+1}A_{N-1,i+1} + q_{N-1,i+1}, \\ &A_{N,N} = A_\infty,\ q_1 = 0, p_1 = 0, \\ &i = 1,2,\dots,\ K_1-1,\ K_1,\ K_1+1,\dots,K_2-1,K_2, \\ &\quad K_2+1,\dots,K_n-1,K_n,\ K_n+1,\dots,\ K_N-1\end{aligned} \tag{10}$$

Solution of non-stationary radon transport equation in many-layered geological medium

Modeling of non-stationary radon transport in geological media is an important tool for solution of problems in radioecology, geophysics and atmosphere physics. Modeling of radon field dynamics in different geological media with further analysis of the obtained time series by method of their comparison with the results of direct and indirect measurements will allow to specify the parameters of radon transport model.

Formulation of the problem

There occurred non-stationary radon transport in geological medium via diffusion and advection processes [5, 6]. It is required to assign distribution of radon volumetric activity along the depth of geological medium and in time. For non-stationary case the equation system (1) for N-layered soil can be written in the following form

$$
\begin{aligned}
&\frac{\partial A_1(z,t)}{\partial t}-\frac{\partial}{\partial z}\left(D_1(z,t)\frac{\partial A_1(z,t)}{\partial z}\right)-\upsilon_1(z,t)\frac{\partial A_1(z,t)}{\partial z}\\
&\quad+\lambda(A_1(z,t)-A_{1,\infty})=0\\
&\frac{\partial A_2(z,t)}{\partial t}-\frac{\partial}{\partial z}\left(D_2(z,t)\frac{\partial A_2(z,t)}{\partial z}\right)-\upsilon_2(z,t)\frac{\partial A_2(z,t)}{\partial z}\\
&\quad+\lambda(A_2(z)-A_{2,\infty})=0\\
&\frac{\partial A_n(z,t)}{\partial t}-\frac{\partial}{\partial z}\left(D_n(z,t)\frac{\partial A_n(z,t)}{\partial z}\right)-\upsilon_n(z,t)\frac{\partial A_n(z,t)}{\partial z}\\
&\quad+\lambda(A_n(z)-A_{n,\infty})=0\\
&\frac{\partial A_{n+1}(z,t)}{\partial t}-\frac{\partial}{\partial z}\left(D_{n+1}(z,t)\frac{\partial A_{n+1}(z,t)}{\partial z}\right)-\upsilon_{n+1}(z,t)\frac{\partial A_{n+1}(z,t)}{\partial z}\\
&\quad+\lambda(A_{n+1}(z)-A_{n+1,\infty})=0\\
&\frac{\partial A_N(z,t)}{\partial t}-\frac{\partial}{\partial z}\left(D_N(z,t)\frac{\partial A_N(z,t)}{\partial z}\right)-\upsilon_N(z,t)\frac{\partial A_N(z,t)}{\partial z}\\
&\quad+\lambda(A_N(z)-A_{N,\infty})=0
\end{aligned}
\tag{11}
$$

At the external borders of many-layered medium the boundary conditions are also assigned

$$\lim_{z\to 0} A_1(z,t)=0,\ \lim_{z\to\infty} A_N(z,t)=A_{N,\infty} \tag{12}$$

and initial condition

$$A_N(z,0)=A_{N,\infty}(z) \tag{13}$$

Method of solution

Balance method is applicable for numerical analysis of the problem (Eqs. (11)–(13)). Let us consider a calculation region for this problem. Let us introduce uniform grids: $\omega_h = \{z_i = ih,\ i = 0,1,2,\ldots,K_1,\ldots,K_2,\ldots,K_N,\ h = 1/K_N\}$ with pitch h on the semi-infinite interval $0 \le z < \infty$, where K_N – rather a large number and $\omega_\tau = \{t_j = j\tau, j = 0,1,\ldots,T,\ \tau = T/K_n\}$ with pitch τ of intercept $0 \le t \le T$. They form overall uniform grid: $\omega_{h\tau} = \omega_h \times \omega_\tau = \{(z_i, t_j),\ z_i = ih,\ 0 < i < K_N,\ t_j = j\tau,\ 0 < j \le T\}$, i.e. the calculation region represents a rectangle. Let us introduce a grid function of solution $A(z_i,t_j) = A_i^j \in \omega_{h\tau}$, which will be considered at integer nodes of uniform grid. Diffusion and advection fluxes will be considered at half-integer nodes. According to methodology mentioned above, we obtain three-diagonal system of algebraic Eqs. (14), after integrating by intercept $z_{i_n-1/2} \le z \le z_{i_n+1/2}$ for n-layer of the soil taking into account time approximation.

$$
\begin{aligned}
&-\alpha_{n,i}A_{n,i-1}^{j+1}+\gamma_{n,i}A_{n,i}^{j+1}-\beta_{n,i}A_{n,i+1}^{j+1}=F_{n,i},\\
&\gamma_{n,i}=\frac{\tau(D_{n,i_n+1/2}^{j+1/2}+D_{n,i_n-1/2}^{j+1/2})}{h^2}+\frac{(\upsilon_{n,i_n+1/2}^{j+1/2}-\upsilon_{n,i_n-1/2}^{j+1/2})}{2h}\\
&\quad+1+\lambda\tau,\\
&F_{n,i_n}=A_{n,i_n}^{j}+\lambda\tau A_{n,\infty i_n}
\end{aligned}
\tag{14}
$$

The solution of the system (14) can be obtained by the double sweep method. Applying this methodology to other layers, we obtain the solution

$$
\begin{aligned}
&A_{1,K_1}^{j+1}=p_{1,K_1+1}A_{1,K_1+1}^{j+1}+q_{1,K_1+1}\\
&A_{2,i}^{j+1}=p_{2,i+1}A_{2,i+1}^{j+1}+q_{2,i+1},\\
&A_{n,i}^{j+1}=p_{n,i+1}A_{n,i+1}^{j+1}+q_{n,i+1}\\
&A_{n,K_n}^{j+1}=p_{n,K_n+1}A_{n,K_n+1}^{j+1}+q_{n,K_n+1},\\
&A_{n+1,i}^{j+1}=p_{n+1,i+1}A_{n+1,i+1}^{j+1}+q_{n+1,i+1}\\
&A_{N-2,i}^{j+1}=p_{N-2,i+1}A_{N-2,i+1}^{j+1}+q_{N-2,i+1},\\
&A_{N-2,K_{N-2}}^{j+1}=p_{N-2,K_{N-2}+1}A_{N-2,K_{N-2}+1}^{j+1}+q_{N-2,K_{N-2}+1},\\
&A_{N-1,i}^{j+1}=p_{N-1,i+1}A_{N-1,i+1}^{j+1}+q_{N-1,i+1},\\
&A_{N,N}=A_{N,\infty i},\ q_1=0,\ p_1=0,\\
&i=1,\ 2,\ldots,\ K_1-1, K_1,\ K_1+1,\ldots,K_2-1, K_2,\\
&K_2+1,\ldots,K_n-1, K_n,\ K_n+1,\ldots,\ K_N-1
\end{aligned}
\tag{15}
$$

Based on the described numerical model of radon transport in non-uniform geological media, the algorithm was developed and program SimRaTran [7] was created. This program allows to model radon transport in porous media represented by several (up to 20) emanating layers with different parameters. It also allows to calculate the distribution of radon volumetric activity and radon flux density along the depth, and radon flux density from the ground surface into atmosphere.

The results of numerical modeling

Let us consider one of the practical problems to solve which we can apply the numerical model (Eq. (10)) described above. The assessment of influence of uranium-containing rocks deposited at a lower depth on the value of near-surface radon activity (at depth of measurement,

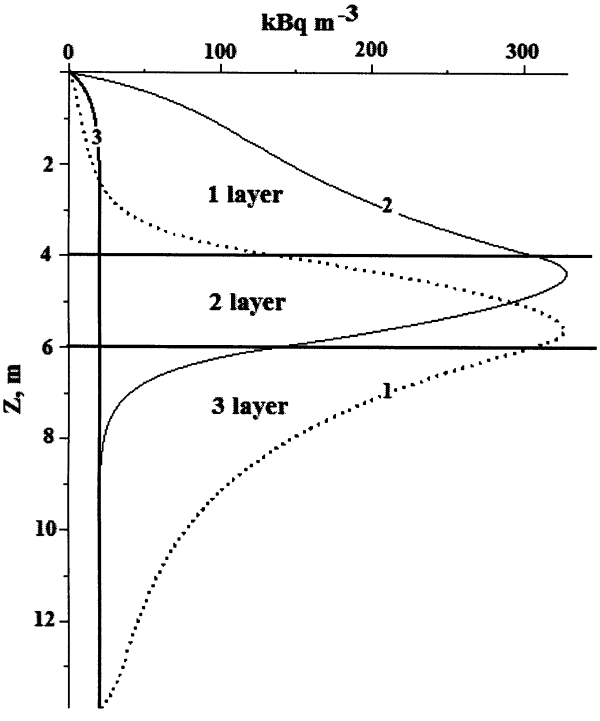

Fig. 2. Radon volumetric activity distribution along the depth of geological medium with three layers: 1 – $\upsilon = 10^{-6}$ m·s^{-1}; 2 – $\upsilon = -10^{-6}$ m·s^{-1}; 3 – uniform medium, $\upsilon = 10^{-6}$ m·s^{-1}.

which is usually ≤ 1 m) is of great practical concern. Let us perform the calculations of the function of radon volumetric activity distribution along the depth for the area whose geological structure is layered.

Let us consider a medium consisting of three layers with a thickness of l_1 = 4 m, l_2 = 2 m and l_3 = 8 m, whose parameters (D_n, $K_{n,em}$, $\rho_{n,s}$, η_n) are similar for analysis simplification. In the first and third layers A_{Ra} = 30 Bq·kg^{-1}. In the second layer the specific activity of radium, which is uranium decay product, was chosen A_{Ra} = 1000 Bq·kg^{-1}, that is typical of uranium-containing rocks.

Numerical calculations were performed at different values and direction of advective velocity, taken in the range of values obtained in work [11]. At positive values of velocity υ, the advective flux is directed towards the ground surface and is summed with diffusion flux, increasing the total radon flux into the atmosphere. At negative values of velocity υ, the advective flux is directed downward from the ground surface decreasing the total radon flux into the atmosphere, and, respectively, the value of radon volumetric activity near the ground surface decreases.

Figure 2 represents the dependence functions of radon volumetric activity upon the depth at positive and negative velocity of advection $|\vec{\upsilon}|$ = 10^{-6} m·s^{-1}. The function of volumetric activity distribution along

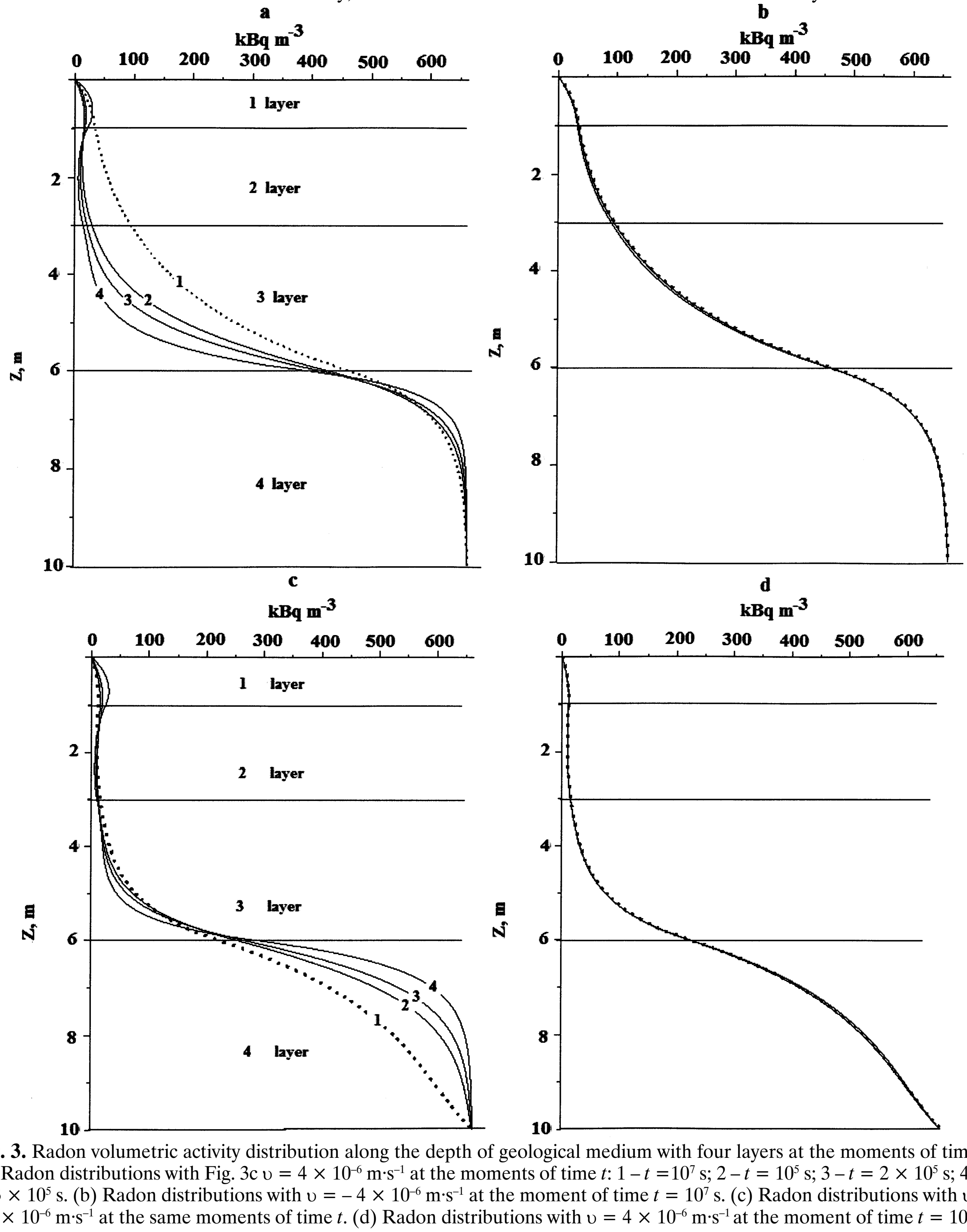

Fig. 3. Radon volumetric activity distribution along the depth of geological medium with four layers at the moments of time *t*. (a) Radon distributions with Fig. 3c υ = 4 × 10^{-6} m·s^{-1} at the moments of time *t*: 1 – *t* = 10^7 s; 2 – *t* = 10^5 s; 3 – *t* = 2 × 10^5 s; 4 – *t* = 3 × 10^5 s. (b) Radon distributions with υ = – 4 × 10^{-6} m·s^{-1} at the moment of time *t* = 10^7 s. (c) Radon distributions with υ = – 4 × 10^{-6} m·s^{-1} at the same moments of time *t*. (d) Radon distributions with υ = 4 × 10^{-6} m·s^{-1} at the moment of time *t* = 10^7 s.

the depth for the uniform medium is shown in the figure for comparison, i.e. all three layers are represented by a loam with a similar radium content A_{Ra} = 30 Bq·kg^{-1}, the advective velocity is assigned as 10^{-6} m·s^{-1}.

Analysis of the functions of radon volumetric activity distribution (Fig. 2) has shown that the influence of the second highly active layer, deposited even at a shallow depth of 4–6 m, can have no impact on the radon measurement results under certain circumstances. At the negative advection velocity, the radon volumetric activity at a depth of 2 m or less does not exceed the values observed in relatively uniform medium.

Figure 3 represents the distribution curves of radon volumetric activity in layered medium (n = 4) at different moments of time t. The thickness of layers is: l_1 = 1 m; l_2 = 2 m; l_3 = 3 m; l_4 = 4 m. For analysis simplification, similar physico-geological parameters for each layer were taken, except radium content which was 90 Bq·kg^{-1} for the first layer, 4 Bq·kg^{-1} – for the 2nd layer, 30 Bq·kg^{-1} – for the 3rd layer and 1000 Bq·kg^{-1} – for the 4th layer.

Figure 3a represents the family of calculation distribution curves of radon volumetric activity in geological medium with the advection velocity of $\upsilon = 4 \times 10^{-6}$ m·s^{-1}. The figure shows that the radon volumetric activity at different moments of time is:

– A = 25–50 kBq·m^{-3} – at the border of the 1st and 2nd layers;
– A = 25–100 kBq·m^{-3} – at the border of the 2nd and 3rd layers;
– A = 400–450 kBq·m^{-3} – at the border of the 3rd and 4th layers.

Figure 3c represents the family of calculation distribution curves of radon volumetric activity in the medium with negative advection velocity $\upsilon = -4 \times 10^{-6}$ m·s^{-1}. Radon volumetric activity has been decreased to the value of:

– A = 10–25 kBq·m^{-3} – at the border of the 1st and 2nd layers;
– A = 25 kBq·m^{-3} – at the border of the 2nd and 3rd layers;
– A = 200–250 kBq·m^{-3} – at the border of the 3rd and 4th layers.

This effect confirms the fact that total radon flux decreases when advection velocity is negative.

Figures 3b and 3d represent the cases, when within a rather long time interval ($t = 10^7$ s) the curves calculated by Eq. (15) obtained from non-stationary model solution coincide with stationary regime of radon transport (Eq. (10)). This validates the developed numerical solution (15). Thus, if the measurements are performed at those periods of time when the advective flux is directed deep into the ground, then the erroneous results of potential radon risk assessment of the examined area can be obtained. And *vice versa*, at large advective fluxes into the atmosphere, radon volumetric activity at the depth of measurement (usually 1 m) can increase by 5 times.

Conclusion

A numerical model of non-stationary diffusion-advection radon transport in many-layered geological media has been developed and its solution has been presented. The peculiarity of radon transport in geological medium has been taken into account in the developed model. This peculiarity is connected with the transport equation coefficients which can be discontinuous at the borders of each layer.

The validity of the developed numerical solution has been confirmed by calculations data using non--stationary numerical model. These data show that at certain moments of time the radon distribution curves calculated by Eq. (15) obtained from non-stationary model solution coincide with stationary regime of radon transport (Eq. (10)).

The analysis of numerical modeling results, while taking into account that advection velocity can vary over a wide range of values and change its sign [11], indicates the necessity to perform integral measurements of radon activity in the soil air for obtaining valid assessment of potential radon risk of the examined area.

Acknowledgment. The work was carried out with the support of the analytical departmental target RF programme "Development of higher school scientific potential", no. 2.1.1/544.

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