NUKLEONIKA 2011;56(2):171–174 ORIGINAL PAPER

Localized plasma polarimetry based on the phenomenon of normal mode conversion

**Yury A. Kravtsov,
Bohdan Bieg**

Abstract. A new geometry of microwave polarimetric measurements is presented, which realizes a localized plasma polarimetry based on the phenomenon of electromagnetic mode conversion. Such a conversion takes place in the tokamak plasma, in the vicinity of the point where the microwave beam is orthogonal to one of the helical magnetic lines. In distinction to the traditional plasma polarimetry, which deals with the line averaged plasma parameters, the new methodology allows for a measurement of the local values of plasma parameters near the point of orthogonality. This methodology was shown to be very efficient in studies of the solar radio emission and polarization properties of radio waves passing through the Earth ionosphere. In the following the theory of electromagnetic mode conversion is described and conditions of its applicability are analyzed. It is shown that localized polarimetric measurements of plasma parameters in the geometry of the international thermonuclear experimental reactor (ITER) device would require very high electron densities N_e, exceeding 10^{17} cm^{-3}, i.e. thousand times higher than those envisaged in the ITER project.

Key words: plasma • polarimetry • mode conversion

Yu. A. Kravtsov✉
Institute of Physics,
Maritime University of Szczecin,
1/2 Wały Chrobrego Str., 70-500 Szczecin, Poland
and Space Research Institute RAS,
82/34 Profsoyuznaya Str., Moscow 117997, Russia,
Tel.: +48 9 1480 9329, Fax: +48 9 1480 9575,
E-mail: y.kravtsov@am.szczecin.pl

B. Bieg
Institute of Physics,
Maritime University of Szczecin,
1/2 Wały Chrobrego Str., Szczecin 70-500, Poland

Received: 22 June 2010
Accepted: 22 October 2010

The traditional (line-averaged) and localized polarimetry

In the traditional plasma polarimetry in thermonuclear devices the probing beam passes through the central zone of the plasma confined in a toroidal vessel. The information on the line averaged values of the plasma density and the magnetic field along the line of sight may be obtained from the changes in the beam polarization resulting from the Faraday and the Cotton-Mouton effects:

$$\Psi \sim \frac{1}{\omega^2}\int N_e B_{0\|} d\sigma, \quad \Phi \sim \frac{1}{\omega^3}\int N_e B_{0\perp}^2 d\sigma \tag{1}$$

where N_e is the electron density, $B_{0\|}$ and $B_{0\perp}$ are static magnetic field components parallel and perpendicular to the beam, ω is the probing beam frequency, and σ is the arc length along the ray.

In this paper a different geometry of measurements is considered, in which the microwave beam propagates through peripheral regions of the plasma. In distinction to the traditional plasma polarimetry, such an orientation allows for a measurement of the local values of plasma parameters. The method is based on the phenomenon of normal mode conversion in the vicinity of the so-called orthogonality point, where the probing beam is perpendicular to the static magnetic field and mutual transformations of ordinary and extraordinary waves take place. The phenomenon of mode conversion has been extensively investigated since the 1960's, firstly in the context of the solar radio emission (see Refs. [1, 9], and the review paper [8]) and then in the context of polarization conversion in the ionosphere plasma [5] (see also Refs. [2, 6]).

The paper is organized as follows. In section *Simple model of helical magnetic lines in a toroidal vessel* we present a simple model for helical magnetic lines in a plasma with the toroidal configuration. Using this model, we analyze in section *The angle between the ray and helical magnetic line* the behavior of an angle between the ray and the magnetic field, which affects the mode conversion. In section *The circular mode conversion near an orthogonality point* we estimate the coefficient of normal wave conversion in the vicinity of the point of orthogonality, using the quasi-isotropic approximation (QIA) approach. Finally, in section *Limitations for the observation of normal mode conversion in the ITER plasma* we analyze the applicability conditions of the localized plasma polarimetry and we establish the range of plasma parameters which would be measurable by this method.

A simple model of helical magnetic lines in a toroidal vessel

Let us consider the toroidal coordinate system (ρ, θ, ϕ), where ϕ is the toroidal angle (Fig. 1), θ is the poloidal angle, measured relative to the vertical axis x, and ρ is the distance from the circular axis of the toroid, defined by the equations $y^2 + z^2 = R^2, x = 0$ (R is the large radius of the toroid). Cartesian coordinates (x, y, z) of the point on the toroidal surface ρ = const., may be expressed in terms of the toroidal coordinates (ρ, θ, ϕ) in the following way:

(2) $$x = \rho\cos\theta,\quad y = (R + \rho\sin\theta)\cos\phi,\quad z = (R + \rho\sin\theta)\sin\phi$$

We use the toroidal surfaces defined by Eq. (2) as a simple approximate model for magnetic surfaces, assuming that the family of the surfaces ρ = const. is topologically similar to the family of the actual magnetic surfaces.

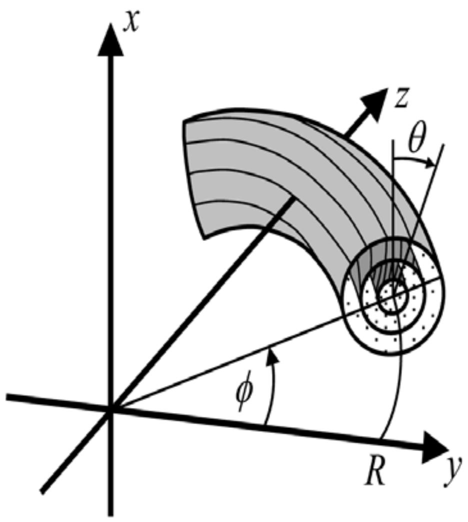

Fig. 1. The toroidal coordinate system (ρ, θ, ϕ).

The helical magnetic lines on the toroidal magnetic surfaces ρ = const. may be described by Eq. (2) supplemented with the notion of spirality, as proposed in [3]

(3) $$\theta = \theta_0 + \phi\chi = \theta_0 + \mu\chi/R,\quad \chi = 1/q,$$

where $\mu = R\phi$ is an arc length along the toroid's circular axis of the radius R and χ is the spirality parameter, which characterizes the rate at which the magnetic line is winding on a toroid: after every full turn around the vertical x axis, the poloidal angle θ increases by $2\pi\chi$. The spirality parameter χ is in fact inversely proportional to the safety factor q, which determines the amount of turns about the vertical axis x, necessary for the closure of the magnetic line. Some of the helical magnetic lines are shown in Fig. 1.

The safety factor q in the real tokamak plasma depends mainly on the distance ρ from the toroid's axis: $q = q(\rho)$. For ITER q is predicted to be about 2–3 in the vicinity of the toroid's axis, increasing slightly on the periphery of the toroid [3].

The angle between the ray and the helical magnetic line

The cosine of the angle $\alpha_{||}$ between the unit ray tangent $\mathbf{l}$ and the unit vector $\mathbf{m}$, tangent to the magnetic line $\mathbf{M}(\sigma)$, is given by the scalar product

(4) $$\cos\alpha_{||} = (\mathbf{m}\cdot\mathbf{l})$$

Let us suppose that the ray lying in the horizontal plane x = const. = h passes through the point $z = 0$, $y = R$ and crosses the axis y at the angle δ, as shown in Fig. 2. The point $x = h, y = R, z = 0$, in which the ray is tangent to the toroidal surface of radius $\rho = h$, will be referred to in what follows as the "reference" point. We suppose that the level $x = h$ does not exceed the minor radius a of the toroidal vessel: $h < a$.

A rectilinear ray $\mathbf{r} = \mathbf{r}(\sigma)$ passing through the reference point is described by the equations $x = h, y = R - \sigma\cos\delta, z = \sigma\sin\delta$, so the unit vector $\mathbf{l} = d\mathbf{r}/d\sigma$ equals $\mathrm{l} = (0, -\cos\delta, \sin\delta)$. Requiring $\cos\alpha_{||}$ to be zero in the reference point and accepting $\theta_0 = 0$ and $\mu = 0$ in this point, we obtain an equation $\tan\delta = h/qR$, that defines the angle δ between the ray and axis y (see Fig. 2).

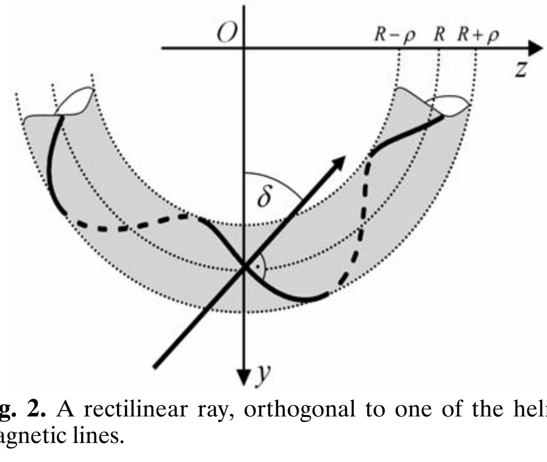

Fig. 2. A rectilinear ray, orthogonal to one of the helical magnetic lines.

The geometry of localized measurements radically differs from the traditional scheme, which relies on rays lying in the poloidal plane ϕ = const. In the localized polarimetry the ray should cross the poloidal plane at an acute angle δ as shown in Fig. 2.

Expressing the toroidal variables through the Cartesian coordinates x, y, z, Taylor-expanding all the functions in Eq. (4) with respect to the ray parameter σ measured from the reference point, and retaining only terms linear in σ we find

$$\cos\alpha_{\|} \approx \frac{\sigma}{\rho_B}, \quad \rho_B \approx \frac{qR^2}{2h} \tag{5}$$

where ρ_B^{-1} characterizes the rate of change of the angle $\alpha_{||}$ in the vicinity of the orthogonality point. An approximate expression $\rho_B \approx qR^2/2h$ is obtained when the large toroid's radius R exceeds the small radius a: $R \geq a$.

The circular mode conversion near an orthogonality point

In what follows we make use of the QIA of the geometrical optics method, which describes propagation of electromagnetic waves in weakly anisotropic plasma [2, 5, 6]. According to [2, 5, 6] the evolution of right and left components of the polarization vector $\Gamma = \Gamma_r\mathbf{e}_r + \Gamma_l\mathbf{e}_l$ in plasma is described by the coupled equations

$$\begin{aligned} \frac{d\Gamma_r}{d\sigma} &= (iG\cos\alpha_{\|})\Gamma_r + (\tfrac{i}{2}GY\sin^2\alpha_{\|})\Gamma_l, \\ \frac{d\Gamma_r}{d\sigma} &= (\tfrac{i}{2}GY\sin^2\alpha_{\|})\Gamma_r + (iG\cos\alpha_{\|})\Gamma_l, \end{aligned} \tag{6}$$

where X and Y are standard plasma parameters:

$$X = \left(\frac{\omega_p}{\omega}\right)^2 \equiv \frac{4\pi e^2 N_e}{m\omega^2}, \quad Y = \frac{\omega_c}{\omega} = \frac{eB_0}{mc\omega} \tag{7}$$

and $G = k_0XY/2$. For high frequency electromagnetic waves used in the plasma polarimetry both parameters X and Y have to be small: $X << 1$, $Y << 1$.

In Eqs. (6) the term $iG\cos\alpha_{||}$ describes the Faraday rotation, whereas the term $i/2GY\sin^2\alpha_{||}$ describes the Cotton-Mouton effect. Under conditions of quasi--longitudinal propagation, when inequality $\cos\alpha_{||} >> Y/2$ holds, the polarization of normal waves is close to the circular one. In this case QIA Eqs. (6) describe the propagation of independent waves of circular polarization. Circular mode conversion takes place when the off-diagonal element on the right hand side of Eqs. (6) becomes comparable with the diagonal element, i.e. when $\cos\alpha_{||} \approx Y/2$. Using a linear approximation (5) for $\cos\alpha_{||}$ and equating diagonal and off-diagonal terms, we can estimate the length of interaction as $l_{\text{int}} \approx Y|\rho_B|$. Due to the inequality $Y << 1$ the interaction length is small in comparison with effective radius ρ_B. When the interaction length is smaller than characteristic scale L_C of the plasma inhomogeneity, which as a rule is comparable to the small tor radius a, i.e. when $l_{\text{int}} << a$, then we may speak about spatial localization of the polarimetric measurements. Accepting linearized expression (5) for $\cos\alpha_{||}$, one may express the solution of the QIA Eqs. (6) in terms of the parabolic cylinder functions [2, 5, 6], and then calculate the coefficient of conversion of the right-handed wave into the left-handed wave: η (right hand $\to$ left hand) = $1 - \exp(-\pi\rho/4)$, where $p = GY|\rho_B| \propto N_eB_0^3$. These results are in complete agreement with the conversion coefficient derived from the method of the phase integrals in [1, 8, 9].

Having measured the transformation coefficient η experimentally, one can find the plasma parameter p from the formula

$$p = -(4/\pi)\ln(1-\eta) \tag{8}$$

Thus, in the case of a spatial localization the coefficient of mode conversion η provides information on the combination of the plasma parameters $N_eB_0^3$ at the point of orthogonality.

Limitations for the observation of normal mode conversion in the ITER plasma

Concerning the limitations inherent to localized polarimetric measurements, there are four points that should be considered.

Firstly, the conversion coefficient η should be "measurable", i.e. it cannot be too small and must be significantly different from unity. Specifically, we assumed $0.05 < \eta < 0.95$. Secondly, the normal modes have to be independent outside the area of interaction. Independence of normal modes implies [4, 7] that beating interval between normal modes is small compared to the characteristic length of plasma inhomogeneity. Thirdly, for the QIA to be applicable the plasma should have the properties of a weakly anisotropic medium. This requirement is met when $Y << 1$. Fourthly, the localization l_{int} of the normal mode conversion area should satisfy the inequality $l_{\text{int}} << a$. Specifically, we have chosen l_{int} = 30 cm, and a = 3 m.

All these four limitations lead to restrictions on the beam frequency and plasma density values that have to be satisfied for the localized polarimetric measurement to be possible. Detailed estimates, based on the plasma parameters typical for ITER, could be expressed by a set of equations [4, 7]:

$$\begin{aligned} N_{1\min}(\omega) &\equiv 4.17 \times 10^{-38}\,\omega^4 < N_e < 2.44 \times 10^{-36}\,\omega^4 \equiv N_{1\max}(\omega) \\ N_{2\min}(\omega) &\equiv 1.1 \times 10^{-12}\,\omega^2 < N_e < 1.8 \times 10^{-21}\,\omega^3 \equiv N_{3\max}(\omega) \\ \omega &> \omega_{\min} = 3.17 \times 10^{13}\ \text{rad·s}^{-1} \end{aligned} \tag{9}$$

and presented in Fig. 3. The range satisfying all limitations (9) is indicated in Fig. 3 as the white area.

The minimal measurable value of the electron density $N_{e\,\min}$ is found to be as high as 10^{17} cm^{-3}. This density is thousand times higher than maximum densities planned for the ITER project. Thus the method of the localized microwave polarimetry, which was found to be very effective in the ionospheric studies, seems to be inapplicable to the toroidal plasma in the ITER device. This analysis also limits the applicability of the localized polarimetry in other thermonuclear devices like JET or W-7X.

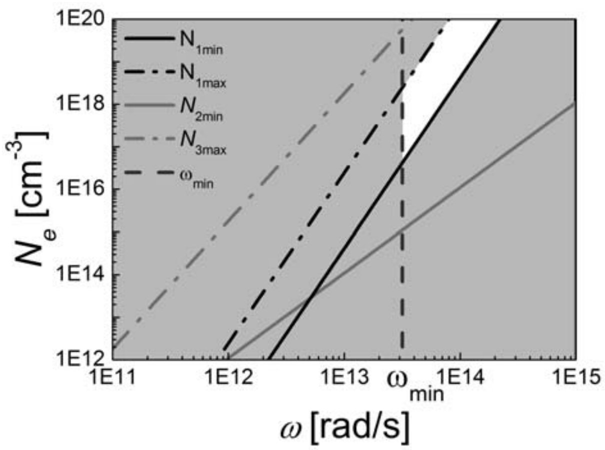

Fig. 3. The plasma densities required for the local measurement.

Conclusions

The method of localized polarimetric measurements deals with the phenomenon of circular mode conversion in the region close to the point of orthogonality between the electromagnetic ray and a helical magnetic line. Theoretically this method allows for a measurement of the product $N_eB_0^3$ near the point of orthogonality, where B_0 the magnetic field. Unfortunately, although this method was proved to be very effective in conditions of the solar and ionosphere plasmas, it would be hardly applicable for measuring plasma parameters in the ITER device. It is shown that the necessary localization of polarimetric measurements on the scale of 30 cm requires very high densities of the order of $N_e = 10^{17}$ cm^{-3}, which significantly exceeds plasma densities $N_e = 10^{13}$–10^{14} cm^{-3}, anticipated for the ITER device.

References

1. Cohen MH (1960) Magnetoionic mode coupling at high frequencies. Astrophys J 131:664–680
2. Fuki AA, Kravtsov YuA, Naida ON (1998) Geometrical optics of weakly anisotropic media. CRC Press, Amsterdam
3. Gormezano C, Sips ACC, Luce TC *et al*. (2007) Chapter 6: Steady state operations. Nucl Fusion 47:S285–S336
4. Kravtsov YuA (2005) Geometrical optics in engineering physics. Alpha Science International, Harrow
5. Kravtsov YuA, Naida ON (1976) Linear transformation of electromagnetic waves in three-dimensional inhomogeneous magneto-active plasma. Sov Phys JETP 44:122–126
6. Kravtsov YuA, Naida ON, Fuki AA (1996) Waves in weakly anisotropic 3D inhomogeneous media: quasi-isotropic approximation of geometrical optics. Physics-Uspekhi 39:129–154
7. Kravtsov YuA, Orlov YuI (1990) Geometrical optics of inhomogeneous media. Springer, Berlin
8. Zheleznyakov VV, Kocharovskiy VV, Kocharovski ViV (1983) Linear coupling of electromagnetic waves in inhomogeneous weakly-ionized media. Sov Phys Uspekhi 26:877–905
9. Zheleznyakov VV, Zlotnik EYa (1964) Polarization of radio waves passing through a transverse magnetic field region in the solar corona. Soviet Astronomy AJ 7:485–491