Isotope effects in chemistry

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Abstract. Isotope effects on chemical equilibria, reaction rates and molecular properties are reviewed together with the theoretical formalism for understanding such effects.

Key words: isotope effects • stable isotopes • kinetic isotope effects

Introduction

The term isotope effect (IE) refers to a difference in some molecular or atomic property consequent to a change in mass or mass distribution caused by isotopic substitution. Radioactive and other specific nuclear effects are excluded. Isotopes of a given element are atoms with the same number of protons in the nucleus (atomic number) but different atomic mass (different number of neutrons). Isotope effects, although generally small, are of interest because within the framework of the Born-Oppenheimer approximation the electronic properties of atoms or molecules can be separated from those involving nuclear motion. The electronic structure determines most of the chemistry and much of the physics. It, in turn, is determined by the charges and charge distributions within the atom or molecule of interest and is nearly mass independent. The solution of the electronic part of the problem defines an electronic potential on which nuclear motion occurs. Thus investigation of isotopic substitution amounts to the determination of how mass and/or mass distribution affects motion on the (isotope independent) potential energy surface. Conversely experimental data on IEs can serve as a tool to gain information about the potential energy surface.

For the heavier elements where relative mass differences between isotopes are small, IEs are correspondingly small. For lighter elements relative mass differences are larger as are the IEs. Some representative values are given in Table 1. IEs on equilibrium constants or vapor pressure amount to a few percent or so for H/D substitution (around room temperature (RT)) but are smaller for the heavier elements ($^{13}\text{C}/^{12}\text{C}$...
or $^{235}\text{U}/^{238}\text{U}$ for example). IEs on rate constants of chemical reaction are typically larger than equilibrium effects (Table 1).

The use of IE studies in the chemical, biological and geological sciences was recently treated by Wolfsberg, Van Hook, Paneth and Rebelo [126]. An important collection of papers treating IEs in chemistry and biology was edited by Kohen and Limbach [65], and Jancso [54] reviewed the field briefly, Melander and Saunders [75], Collins and Bowman [21], Bigeleisen and Wolfsberg [16], and Truhlar [101] have dealt with IEs on chemical reaction rates. Cook [22], Cook and Cieland [23], and Cieland, O’Leary and Northrup [19] discuss IEs on enzyme catalyzed reactions. Hoefs [46] has reviewed stable isotope geochemistry. Criss [25] and Galimov [33] reviewed the field briefly. Melander and Saunders [75], Kohen and Limbach [65], and Jancso [54] to discover the rare isotopes of oxygen, carbon, and nitrogen respectively. However neither the carbon, and nitrogen had isotopes. This was due to the fact that then existing mass spectrographs were too insensitive to detect these isotopes because of their low abundance. A different technique was required. High resolution emission spectroscopic methods in the visible/UV developed by Mulliken [78] were employed by Giauque and Johnston [35, 36], King and Birge [62] and Naude [79] to discover the rare isotopes of oxygen, carbon, and nitrogen respectively. However neither the mass spectrographic nor spectroscopic techniques available at the time permitted characterization of the very low abundance (0.015%) heavy isotope of hydrogen, $\text{H}_2$, known as deuterium, D, even though its existence was discovered for several years. At the time permitted characterization of the very low abundance (0.015%) heavy isotope of hydrogen, $\text{H}_2$, known as deuterium, D, even though its existence was suspected for several years. By the late 1920’s, although Aston and his colleagues had discovered a large number of isotopic atoms, there was no evidence that the lighter elements hydrogen, carbon, nitrogen and oxygen had isotopes. This was due to the fact that then existing mass spectrographs were too insensitive to detect these isotopes because of their low abundance. A different technique was required. High resolution emission spectroscopic methods in the visible/UV developed by Mulliken [78] were employed by Giauque and Johnston [35, 36], King and Birge [62] and Naude [79] to discover the rare isotopes of oxygen, carbon, and nitrogen respectively. However neither the mass spectrographic nor spectroscopic techniques available at the time permitted characterization of the very low abundance (0.015%) heavy isotope of hydrogen, $\text{H}_2$, known as deuterium, D, even though its existence had been suspected for several years. Soddy coined the term “isotopes” to distinguish them. The introduction of the concept of atomic number by Van den Broek [107, 108] and Moseley [77] clarified matters, i.e. showed that isotopes differ in atomic mass, not atomic number. After the hiatus caused by World War I, Lindemann and Aston [68] discussed the possibility of separating a mixture of isotopes by distillation, centrifuge, or by producing positive rays and subjecting them to electric and magnetic fields (the method still employed in the mass spectrograph). Also Lindemann [67] reported the first theoretical analysis of the vapor pressure isotope and applied it to earlier efforts to separate isotopes of neon. These efforts led to Aston’s development of the mass spectrograph and his use of that instrument to determine the relative masses of the isotopes of neon and chlorine to procure his Nobel Prize (Aston [3]). During the 1920’s Aston employed the mass spectrograph to discover no less than 212 of the naturally occurring isotopes. From these results he was able to formulate the whole number rule of atomic structure. Independently, Harkins and Wilson [41–43] working at the University of Chicago made important contributions to the development of the whole number rule. Harkins also worked out the first partial separation of the chlorine isotopes (Harkins and Hayes [40]).

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They observed the expected spectral lines for deuterium from both samples but the evaporative residue was much enriched. No evidence of tritium (\(^{3}\text{H} = ^{3}\text{T}\)) was observed. This work led to the award of the 1934 Nobel Prize to Urey. The discovery of deuterium generated a rush of work on deuterium, so much so that Urey and Teal [106] published a sixty page review of the field just a few years later. It is quite understandable that studies of isotope effects were invigorated by the availability of deuterium since the relative mass difference H to D is so much larger than the heavier elements and provides easily measurable isotopic differences. The spurt in IE studies, however, ended around 1940 with the beginning of World War II and the security classification of almost all work in isotope science.

**Fundamental theory of isotope effects**

The Born-Oppenheimer approximation, molecular vibrations

The interpretation of experimental data on IEs is almost invariably carried out using the Born-Oppenheimer approximation [17] which is employed to determine an isotope independent electronic potential on which nuclear motion occurs. Corrections to this approximation are vanishingly small for atoms heavier than hydrogen, and even there can be ignored except in special circumstances (Bardo and Wolfsberg [4, 5]). The normal starting point considers isolated molecules in the gas phase. Nuclear motion is described in terms of small displacements from the equilibrium positions. The potential energy is approximated with an expansion in terms of displacement coordinates where the leading (quadratic) term is the largest, but cubic and higher terms may contribute. Even so, these higher terms are generally neglected and this leads to a description in terms of a set of \(3n-j\) harmonic frequencies \(n\) is the number of atoms in the molecule, \(j = 5\) for a linear, 6 for a nonlinear, and 3 for a monatomic molecule). The frequencies can be obtained by diagonalization of the matrix \(FG\), where \(F\) is an isotope independent \((3n-j)\) dimensional potential energy (force constant) matrix and \(G\) is a kinetic energy matrix which takes proper account of mass and mass distribution. Given \(F\) and \(G\) for the set of isotopic molecules one can calculate a complete set of frequencies and isotopic frequency shifts. These can then be employed to evaluate partition functions, partition function ratios, and isotope effects on thermodynamic properties using the methods of statistical thermodynamics. The entire procedure is well suited to machine calculation and Wolfsberg et al. [126] give details and appropriate literature references.

In the calculation of thermodynamic properties it is necessary to account for the \(j\) zero-frequency rotations and translations which lie at the classical limit in the gas, but take on nonzero values in the condensed phase. One notes that the isotopic frequency shifts are small, at least for isotopes heavier than hydrogen, and it becomes convenient to formulate the isotopic partition function ratios (which lie near unity) by expansion in powers of \(1/T\). It has been long established that IE’s of the type under discussion vanish in the classical (high temperature) limit and Jacob Bigeleisen and Maria Mayer ([15] 1947, Fig. 1) demonstrated the convenience of defining a reduced partition function ratio, \(q_{\text{quantum mechanical}}/q_{\text{classical}}\). This method has the advantage of focusing attention directly on the isotope effects themselves rather than on small differences in the enormously larger values of the partition functions for each species. The Bigeleisen-Mayer approach has long since been widely employed. Waldmann [119] independently proposed an approach similar to the Bigeleisen-Mayer formulation.

The methods outlined above have been applied to a wide variety of special cases including IEs on chemical equilibria, and IEs on rate constants (using transition state theory), vapor pressure, molar volume, etc. Sometimes corrections for anharmonicity, rotation-vibration interaction, centrifugal distortion, and deviations from the Born-Oppenheimer approximation have been...
applied. Most analyses consider systems in thermal equilibrium and employ Maxwell-Boltzmann distribution functions. Isotope effects for systems not in thermal equilibrium are also of interest. Non-Maxwellian distributions are important in the treatment of laser methods of isotope separation, IEs on fast unimolecular reactions, etc.

Substituent effects and the first law of isotopics

Studies of the changes in properties when one atom in a molecule is substituted by another (e.g. replacing Cl by Br in CH₃Cl) have been a useful tool of chemists for many years. An isotope effect is a special type of substituent effect. The fact that two isotopomers have the same potential energy surface for nuclear motion to excellent approximation means that isotope effects are theoretically accessible if one knows the potential energy surface. Conversely if one does not know the potential energy curve the isotope effect can be used to learn about the surface. Thus, the Born-Oppenheimer isotope independent potential energy surface is central to the understanding of isotope effects and has been referred to as the First Law of Isotopics.

Isotope exchange equilibria

Some examples of IEs on equilibrium constants have been included in Table 1. The Bigeleisen-Mayer [15] treatment introduces the reduced partition function ratio, \((s_2/s_1)f\),

\[
\frac{s_2}{s_1} f = \left(\frac{s_2}{s_1}\right) \left(\frac{q_{2i}}{q_{1i}}\right) \frac{1}{\prod_{i=1}^{N} \left(\frac{m_{2i}}{m_{1i}}\right)^{2/3}}
\]

(1)

\[
= \prod_{i} \frac{u_{2i}}{u_{1i}} \left(1 - e^{-u_{2i}/kT}\right) \left(1 - e^{-u_{1i}/kT}\right)^{-1/2}
\]

(2)

Subscript 1 refers to the more lightly substituted isotope. The definition recognizes that the product of isotopic masses always cancels in the calculation of the IE on the equilibrium constant. It is the same on both sides of the reaction. In Eq. (1) \(u_i = h\nu c/kT\) and the product is over the \(3n-6\) (3\(n-5\) if the molecule is linear) normal mode harmonic oscillator frequencies. In these equations \(s_2\) and \(s_1\) are symmetry numbers and cannot lead to any isotope effect. The units of \(\nu\) are \(cm^{-1}\) and \(c\) is the speed of light. Equation (2) points out the RPFR can be considered as the product of three factors: the product factor (PF), the excitation factor (EXC) and the zero point energy contribution (ZPE). Throughout, the energy zeros are taken at the bottom of the harmonic oscillator wells.

As an example consider the replacement of H by D

\[
AH + B = CH + E\quad K_1
\]

(3)

\[
AD + B = CD + E\quad K_2
\]

(4)

so

\[
\frac{K_1}{K_2} = \left(\frac{q_{2D}}{q_{1D}}\right) \left(\frac{q_{2C}}{q_{1C}}\right)
\]

(5)

since \(A, B, C\) and \(E\) are identical in the two reactions and from the Born-Oppenheimer approximation the two reactions share a common zero. Using Eqs. (1) through (5)

\[
AH + CD = AD + CH\quad K_3
\]

(6)

\[
\frac{K_3}{K_1} = \frac{K_3}{K_2} = (\text{symmetry number factor})
\]

(7)

\[
\frac{\left(\frac{s_2}{s_1}\right)f\left([AD/\text{AH}]\right)}{\left(\frac{s_2}{s_1}\right)f\left([CD/\text{CH}]\right)}
\]

where the symmetry number factor is \((s_{AD}/s_{AH})(s_{CD}/s_{CH})\). The IE is the product of the symmetry number factor and terms which depend only on normal mode vibrational frequencies. The introduction of the RPFR had a profound effect on IE studies. Equation (1) focuses attention of the isotope dependent frequencies and the isotope independent force constants which define those frequencies. Isotope effects reflect force constant differences at or near the position of isotope substitution on the two sides of the reaction.

Limiting values: the low and high temperature limits

At low temperature \(u_i\) increases, \((1 - e^{-u_i}) \rightarrow 1\) and RPFR = \((s_2/s_1)f = (PF)(EXC)(ZPE) \rightarrow \prod (u_{2i}/u_{1i}) (\exp[-(u_{2i} - u_{1i})/2])\). No upper vibrational levels are populated; this condition is known as the zero point energy approximation, or, alternatively, as the low temperature approximation. For a typical \(CH\) stretching frequency, \(\nu \sim 3000\ \text{cm}^{-1},\ u \sim 4\) at 1050 K and it is reasonable to use the ZPE approximation for that frequency at temperatures below 1000 K. To the same precision for \(\nu \sim 300\ \text{cm}^{-1},\ u \sim 4\) around 100 K and the ZPE approximation is only useful below that temperature. Note that at very low temperature IEs can get very large.

At high temperature \(u_i = h\nu c/kT\) and in Eq. (1) ZPE = \(e^{u_i^2} \rightarrow e^{\nu^2} = 1\). Also \(1 - e^{-u_i} \sim (1 - u + u^2/2 + ... = u\), so

\[
\lim_{x \rightarrow 0} \prod \frac{u_{2i}}{u_{1i}} \left(1 - e^{-u_{2i}/kT}\right) \left(1 - e^{-u_{1i}/kT}\right) \rightarrow 1
\]

(8)

and, overall, \((s_2/s_1)f\) approaches unity. Thus at high temperature the limiting equilibrium constant IE, Eq. (7), is given by a ratio of symmetry numbers. The symmetry number ratio is a purely statistical factor and will not lead to any enrichment of isotopes. There are no isotope effects in the classical high temperature limit.

One can explore the approach to the high temperature limit by carrying additional terms in the expansions of \(e^{u^2}\) and \(1 - e^{u^2}\), in this fashion obtaining...
Thus, the first order correction to classical (high temperature) statistical mechanics is proportional to $\hbar^2$ (Wigner [122]).

An example, $H_2 + 2DI = D_2 + 2HI$

The vibrational frequencies needed to calculate this exchange equilibrium constant are given in Table 2. To good enough approximation $u_2 = u_1/\sqrt{2}$ for both the $H_2/D_2$ and the HI/DI pairs.

$$K = \left[\frac{(s/s')f(H_2/D_2)}{(s/s')f(HI/DI)}\right]^2$$

In each case the prime refers to the lighter isotope. At all but very high temperatures it is necessary to employ the complete equation because the vibrational frequencies for all four molecules are quite high. At room temperature $u(H_2) \approx 21$ and $u(HI) \approx 11$. The results of the calculation are shown in Fig. 2. At very high temperature Eq. (9) may be used and the exchange constant is

$$K = 1 + \left(\frac{1}{24}\right)\left(\frac{\hbar}{kT}\right)^2 \sum (v_i^2 - v_{i+1}^2)$$

This result is shown in Fig. 2. More precise calculations include corrections for nonclassical rotation and vibrational anharmonicity. The corrections lower the calculated isotope effects by only several percent. The calculated IEs are in good agreement with experiment.

**Corrections to the Bigeleisen-Mayer equation. The nuclear field shift effect**

Several corrections to the Bigeleisen-Mayer equation need to be considered. The most important is the correction for anharmonicity (Wolfsberg [125]). Anharmonic zero point corrections are only important for vibrations involving light atoms. They account for changes of at most 1% or so in logarithmic fractionation factors calculated for exchange between molecular species containing H, D or T. Two other corrections arise from the use of the Born-Oppenheimer approximation. The first, the adiabatic correction (Bardo and Wolfsberg [4, 5]), arises from the coupling between electronic and nuclear motion and, again, is only important for reactions involving hydrogen isotopomers. It is completely negligible for heavier atoms because it scales proportionally to $\delta M/M^2$. The second arises from a shift in electron energy states in an atom or molecule due to the perturbation of those electrons with high charge density at the nucleus. This, the field shift correction, depends on nuclear size and shape. Since the electron is bound more strongly to the smaller (lighter) nucleus with its higher charge density, the ground state of the heavier isotopomer will lie higher than the one for the lighter isotopomer. This is opposite to the ordering of the zero point energies associated with molecular vibrations, but until the late 1980’s it was not suspected that the effects were large enough to affect thermodynamic isotope effects. Fuji and coworkers [31], however, reported anomalies in separation factors for the (uranium-IV/uranium-VI) red-ox exchange reaction between $^{238}$U and lighter uranium isotopes. Interestingly the separation factors for odd/even ($^{233}$U/$^{238}$U and $^{235}$U/$^{238}$U) pairs lie significantly above the correlation lines for the even/even ($^{234}$U/$^{238}$U and $^{236}$U/$^{238}$U) pairs. According to the Bigeleisen-Mayer formalism all the separation factors should scale proportionally to the mass difference ($^{238}$U-$^i$U). The anomaly lies well outside the experimental error. It was Bigeleisen [12] who demonstrated the ($^i$U/$^{238}$U) separation factors can be written as the sum of vibrational (Bigeleisen-Mayer) and field shift contributions. He theoretically calculated the isotope independent scaling factor which defines the mass dependence of the field shift term thus explaining the anomaly. It is interesting and important to recognize that the field shift and Bigeleisen-Mayer

**Table 2. Spectroscopic properties needed to calculate $K_{eq}$ for $H_2 + 2DI = D_2 + 2HI$**

<table>
<thead>
<tr>
<th>Molecule</th>
<th>$\omega_e$/cm$^{-1}$ (a)</th>
<th>$\omega_{ex}$/cm$^{-1}$ (b)</th>
<th>$B_0$/cm$^{-1}$ (b)</th>
<th>$\sigma T$ (b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>H$_2$</td>
<td>4405.3</td>
<td>125.3</td>
<td>59.3</td>
<td>85.4</td>
</tr>
<tr>
<td>D$_2$</td>
<td>3117.1</td>
<td>63.0</td>
<td>29.9</td>
<td>43.0</td>
</tr>
<tr>
<td>HI</td>
<td>2309.5</td>
<td>39.7</td>
<td>6.5</td>
<td>9.3</td>
</tr>
<tr>
<td>DI</td>
<td>1640.2</td>
<td>20.0</td>
<td>3.3</td>
<td>4.7</td>
</tr>
</tbody>
</table>

(a) Harmonic frequency.
(b) Parameters required for anharmonic corrections. E.g. Wolfsberg [125].

**Fig. 2.** Harmonic oscillator rigid rotor equilibrium constant for $H_2 + 2DI = D_2 + 2HI$. This graph shows the harmonic oscillator rigid rotor equilibrium constant, $K$, for $H_2 + 2DI = D_2 + 2HI$, with the comparison between complete and high temperature approximations.
contributions are of opposite sign. For the uranium system the field shift makes the largest contribution to the separation factor. The vibrational contribution leads to a preference for the heavier isotope in the U(VI) species, while the larger field shift effect prefers the heavy isotope in the U(IV) species. The net effect is a preference of the heavy isotope in U(IV). The anomalous odd/even behavior is due to the fact that the $^{233}$U and $^{235}$U isotopomers have nuclear quadrupole moments, the even mass isotopomers do not. Consequently the field shift does not scale proportionally to mass difference. Field shift contributions to redox separation factors have been observed for other metal systems. They only need to be considered for cases involving atoms with high nuclear charge densities (heavy metals).

**Kinetic isotope effects**

**Transition state theory of isotope effects**

The basic idea employed in the transition state theory of chemical reactions as developed by Eyring and others is diagrammed in Fig. 3 (Glasstone, Laidler and Eyring [37], Bigeleisen and Wolfsberg [16], Van Hook [110], Wolfsberg et al. [126]). The reaction takes place on a $3n$ dimensional isotope independent electronic potential energy surface ($n$ the total number of atoms in the system). One of these dimensions, the reaction coordinate, corresponds to the minimum energy path between the minima characterizing reactant and product. The transition state lies in a saddle point at the maximum, $\Delta E^\ddagger$, of this minimum energy path. Energies are measured from the potential energy minima. The reaction can proceed semi-classically along the reaction coordinate, over the barrier and its transition state, to product, or, alternatively by quantum mechanical tunneling through the barrier with (generally) smaller probability. Note that the curvature along the reactant coordinate is negative. Motion along this coordinate will be treated separately.

We now consider a chemical reaction

$$A + B \rightarrow E + F$$

(12)

The rate constant, $k$, is defined by the relation $-d[A]/dt = k[A][B]$ and is not to be confused with the Boltzmann constant, $k$ (or $k_B$ where the use of “$k$” would lead to confusion). The square brackets denote concentrations. A central assumption of transition state theory is that equilibrium is maintained between reactants and transition state. Thus the rate of reaction is given by the equilibrium concentration of transition state $[AB^\ddagger]$ multiplied by the rate of decomposition of the transition state, $r$, and by $\kappa$ the probability of motion in the forward direction (to products) rather than reverse (to reactants).

$$\frac{d[A]}{dt} = r K^i [A][B]$$

(13)

where $K^i = [AB^\ddagger]/([A][B])$ is the equilibrium constant between reactants and transition state. The factor $r$ is evaluated using the one dimensional translational partition function which treats motion along the reaction coordinate. One obtains

$$k = \frac{k_B T}{h} K^i \kappa$$

(14)

where $K^i$ has been replaced with $K^i\ddagger$ to indicate these later expressions for the partition function now contain $3n^\ddagger – 7$ ($3n^\ddagger – 6$ if linear) real vibrations. Of course the reactant partition functions are treated using $3n^\ddagger – 6$ real vibrations for each $A$ and $B$ species. The parameter $\kappa$ (which may be significantly large) is dominated by tunneling. A brief discussion of tunnel effects is deferred to a later section. The isotopic rate effect on the rate constant is

$$\frac{k_1}{k_2} = \frac{K^i_1 \kappa}{K^i_2 \kappa}$$

(15)

with 1 and 2 referring as usual to isotope substitution, say $A_1$ and $A_2$ and their corresponding transition states, $AB^\ddagger_1$ and $AB^\ddagger_2$. Introducing partition functions for reactants and transition states

$$\frac{K^{i\ddagger}_1}{K^{i\ddagger}_2} = \frac{(q_{A_1} q_{A_1}^*) / (q_{A_1} q_{B_1}^*)}{(q_{A_2} q_{A_2}^*) / (q_{A_2} q_{B_2}^*)} = \left(\frac{q_{A_1} / q_{A_1}}{q^{*}_{A_2}} / q_{A_2}^* / q_{A_2}^*\right)$$

(16)

The reduced partition function ratio for the transition state is then straightforwardly,

$$\left(\frac{s_2}{s_1}\right)^i f^i = f^i = \prod_n \frac{u_{i}^{(1)}}{u_{i}^{(2)}} \left(1 - e^{-\beta q_i^{(1)}}\right) \left(1 - e^{-\beta q_i^{(2)}}\right)$$

(17)

Because the reaction coordinate has been treated separately the product is over $3n^\ddagger – 7$ rather than $3n^\ddagger – 6$ coordinates. For nonlinear transition states there are $3n^\ddagger – 6$ coordinates. The ratio of isotopic rate constants is
Modern methods to calculate RPFRs, isotope equilibrium constants, and kinetic isotope effects

Spectroscopic data are available for a large number of molecules, including isotopically labeled molecules, and it would appear straightforward to employ such data to calculate \( \frac{s_k f}{s_i k} \) for reactants and products in chemical equilibria and rate expressions. This has proved to be unsatisfactory. Except for the simplest molecules some normal mode frequencies are spectroscopically forbidden or not observed for other reasons. Also the precision of vibrational frequency measurements is often insufficient to permit calculation of RPFR at useful precision. Moreover observed frequencies are anharmonic and the corrections needed to produce sets of harmonic frequencies are troublesome. For these reasons it is customary to employ the experimental frequencies to deduce a set of isotope independent force constants which, on the one hand, are consistent with the spectroscopic data, and on the other can be employed to calculate a self consistent set of harmonic frequencies for all isotopomers, and thence high precision isotope frequency shifts and RPFRs. Typically some variant of the Wilson, Decius, Cross [123] FG method is employed for this purpose. This approach is well suited for modern computational methods and computer programs for that purpose were introduced by Schachtsneider and Snyder [88], then adapted to IE studies by Stern and Wolfsberg [96] who made a thorough study of the relationship between force constant changes (reactant to product, or reactant to transition state) and the calculated isotope effects. More recently computations have moved back a step further toward fundamental theory. They begin with an \textit{ab initio} quantum mechanical calculation of the Born-Oppenheimer surface (Wolfsberg et al. [126]), Cramer [24]. The Born-Oppenheimer surface is probed in the vicinity of the minima and saddle points to establish gradients and force constants for the reactant(s) and transition state. This involves the calculation of the energy at numerous points along each coordinate and is computationally intensive. One commonly used program is Gaussian03 but a number of other programs are available. The details are highly technical and will not be explored in this review. Many examples of theoretically calculated isotope effects described in this or other reviews or cited in the tables have employed these general procedures.

Tunneling

It is well understood that a chemical reaction can occur even for a system whose energy is less than the barrier height. This purely quantum mechanical phenomenon is referred to as tunneling. The probability of tunneling decreases as the energy deficit for classical passage increases and as the length of the tunneling path increases. Also light particles have much higher tunneling probability than do heavier ones. As it turns out then, tunneling is most important for the transfer of light particles (H, H\textsuperscript{+} or H\textsuperscript{−} vs. D, D\textsuperscript{+} or D\textsuperscript{−}) through narrow barriers. Kohan [64] has described the observational criteria generally used to determine whether or not tunneling is an important factor in determining isotope effects on any particular reaction or reaction sequence.

In both TST and VTST tunneling is introduced as a correction factor, generally denoted \( \kappa \). In TST tunnel corrections are sometimes calculated using the Wigner \( u^2/24 \) law [122], see Eq. (9).

\[
\kappa = 1 - u^2/24 + \ldots
\]

In this equation \( u^2 \) is the reduced (imaginary) frequency which describes barrier curvature. Much better and much more widely used approximations were introduced...

 Variational transition state theory

In spite of the many successes of transition state theory (TST) it is inadequate for a proper treatment of reaction kinetics. Ordinary TST is not capable of correctly describing quantum effects and barrier penetration (tunneling). A more general theory, variational transition state theory (VTST), developed by Truhlar and coworkers [101], Allison and Truhlar [2] optimizes the location of the surface dividing reactants from products, placing it so that forward flux is minimized. The basic idea is to find a dividing surface which leads to a higher free energy of activation and thus a lower rate constant. In TST the dividing surface is defined geometrically as the saddle point on the Born-Oppenheimer surface, but in VTST it is that surface which leads to the minimum value of the rate constant. In considerations of isotope effects the potential surface is isotope independent in both TST and VTST. In TST the transition state for both isotopes is at the saddle point, but for VTST the transition point need not be (and usually is not) at the saddle point, and may be different for the two isotopes. As a consequence the expression for the isotope effect on a rate constant \( k_i/k_2 \) must include a factor involving the energy differences of the isotopic dividing surfaces. Additionally, the force constants from which the vibrational frequencies of the isotopic transition states are calculated, and the geometries of the transition states, may differ a bit. These points reemphasize the fact that TST is much simpler conceptually than VTST. There is just one transition state in TST and it is located at the saddle point on the minimum energy path. In VTST the dividing surface for each isotope is temperature dependent (since the partition functions and consequently the free energy of activation are temperature dependent).

\[
\frac{k_1}{k_2} = (\text{symmetry number factor})
\]

\[
\times \sqrt{\frac{s_k f}{s_i k}} \left( \frac{A_z^2}{A_2^2} \right) \kappa_i \times \sqrt{\frac{s_k f}{s_i k}} \left( \frac{A_z B_z^2}{A_2 B_2^2} \right) \kappa_2
\]

As before, symmetry number factors do not lead to isotope enrichment.

The use of the RPFR to study kinetic isotope effects was first undertaken by Bigeleisen [10], this work was elaborated by Bigeleisen and Wolfsberg [16]. It has since been applied to a plethora of reactions.
by Bell (parabolic barrier) and by Eckart. The Bell and Eckart expressions each lead to exactly soluble quantum mechanical expressions for the barrier reflection and transmission probabilities. Details are available elsewhere (Wolfsberg et al. [126], Melander and Saunders [75]).

In the complete VTST calculations the tunneling factor is set equal to the ratio of the temperature averaged quantum transmission coefficient and the corresponding classical transmission coefficient. The quantum mechanical tunneling problem can be solved exactly for the three atom collinear case (and even for the corresponding three dimensional case) but the general problem is still very difficult mathematically. Marcus and Coltrin [73] and Truhlar [101] describe calculations of multidimensional tunneling factors which yield reaction probabilities in excellent agreement with the exact quantum mechanical values. An example is given in Fig. 4.

**Tests of VTST (including tunneling)**

Table 3 shows comparisons of TST and VTST calculations of IEs for collinear reactions with the exact quantum mechanical calculations. Table 4 compares experimental and VTST calculated values for D + H₂ = (D–H–H)‡ = DH + H and H + D₂ = (H–D–D)‡ = HD + D. The excellent agreement between theory and experiment is gratifying. Table 5 compares experimental and calculated IEs for some enzyme reactions. Tables 3, 4 and 5 clearly demonstrate that the VTST theory is superior to TST especially for light atom transfer. Nonetheless the fact remains that TST is adequate to rationalize the basic features of the isotope effects. For that reason it remains in widespread use for interpretation of experimentally measured isotope effects. This is because the computational complexity of VTST often outweighs its practical implementation. VTST calculations are far more tedious and expensive, and VTST packages for computer implementation are not so widely available as those for TST.

**Kinetic isotope effects continued**

**Primary and secondary IEs**

Primary kinetic isotope effects are those in which the bond to the isotopically substituted atom is broken (or otherwise directly changed) in the course of the reaction.

**Table 3.** Tests of TST and VTST by comparing with exact quantum calculations; isotope effects at 300 K. The numbers in the table are ratios of rate constants for the two selected reactions (Allison and Truhlar [2]).

<table>
<thead>
<tr>
<th>Reaction</th>
<th>k_D/k_H</th>
<th>TST</th>
<th>Best VTST</th>
<th>Exact</th>
</tr>
</thead>
<tbody>
<tr>
<td>H + HH' → HH + H</td>
<td>1.33</td>
<td>3.85</td>
<td>4.35</td>
<td></td>
</tr>
<tr>
<td>D + HH → DH + H</td>
<td>0.62</td>
<td>0.60</td>
<td>0.57</td>
<td></td>
</tr>
<tr>
<td>H + DD → HD + D</td>
<td>1.33</td>
<td>1.56</td>
<td>1.61</td>
<td></td>
</tr>
<tr>
<td>D + DD → DD + D</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>H + FF → HF + F</td>
<td>7.25</td>
<td>6.27</td>
<td>5.37</td>
<td></td>
</tr>
<tr>
<td>D + FF → DF + F</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cl + HCl → CIH + Cl</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cl + DCI → CID + Cl</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table 4.** Temperature dependence of the hydrogen isotope reaction, D + HH → DH + H and H + DD → HD + D

<table>
<thead>
<tr>
<th>T (K)</th>
<th>k_DHH/k_HHD</th>
<th>Calculatedab</th>
<th>Experimentab</th>
</tr>
</thead>
<tbody>
<tr>
<td>200</td>
<td>74</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>16</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>7.9</td>
<td>7.6</td>
<td></td>
</tr>
<tr>
<td>800</td>
<td>2.7</td>
<td>2.8</td>
<td></td>
</tr>
</tbody>
</table>

a Mielke et al. [76].
b Ridley et al. [86].
Isotope effects in chemistry

Secondary kinetic isotope effects are those in which the isotope substitution occurs at position(s) away from the bonds being made or broken. For example in the second order SN$_2$ reaction of hypochlorite and ethyl chloride

\[
\text{ClO}^- + \text{CH}_3\text{CH}_2\text{Cl} \rightarrow \text{ClOCH}_2\text{CH}_3 + \text{Cl}^- \]

the isotopic ratio $[k(35\text{Cl})/k(37\text{Cl})]$ corresponds to a primary chlorine isotope effect, while $[k(\alpha-\text{H}_2)/k(\alpha-\text{D}_2)]$ corresponds to an $\alpha$(alpha) deuterium secondary isotope effect ($\alpha = 1$ bond between the position of isotope substitution and the reacting bond, $\beta = 2$ bond separation, etc.). Secondary IEs have been reviewed by Hengge [44]. Typically they are markedly smaller than primary effects and are often inverse. They fall off in the order $\alpha > \beta > \gamma$, etc. Secondary IEs for $\text{S}_\text{N}2$ reaction of hypochlorite and ethyl chloride

\[
(20) \quad \text{ClO}^- + \text{CH}_3\text{CH}_2\text{Cl} \rightarrow \text{ClOCH}_2\text{CH}_3 + \text{Cl}^- \]

approximately described by the Swain-Schaad equation (Swain et al. [98])

\[
(21) \quad (k_{\text{H}}/k_{\text{T}}) = (k_{\text{H}}/k_{\text{D}})^{1.44} \]

In deriving Eq. (21) one assumes the H, D and T motions can all be treated in the ZPE approximation and the only important isotope sensitive motions are the stretching modes. These drastic assumptions imply the result is can only be used for qualitative purposes. Hirschi and Singleton [45] have discussed deviations from the Swain-Schaad relation for reactions without tunneling or kinetic complexity.

Isotope effects as probes for reaction path and/or transition state structure

It has long been argued that IE studies provide insight into the structure of the transition state, a property not amenable to direct experimental observation. Two examples chosen from the multitude available in the literature follow.

Figure 5 illustrates competitive reaction paths for the reaction of hypochlorite anion with ethyl chloride (Villano et al. [117]).

Relative isotope effects: the Swain-Schaad relation

It is often useful to make comparisons of KIEs for multiple pairs of isotopes of the same element, say H/D vs. H/T. The relationship between $k_\text{H}/k_\text{D}$ and $k_\text{H}/k_\text{T}$ is

\[
(21) \quad (k_{\text{H}}/k_{\text{T}}) = (k_{\text{H}}/k_{\text{D}})^{1.44} \]

Table 5. Experimental and calculated isotope effects for several enzyme systems$^a$

<table>
<thead>
<tr>
<th>Enzyme</th>
<th>KIE</th>
<th>TST</th>
<th>Best VTST</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Enolase</td>
<td>Primary</td>
<td>4.7</td>
<td>3.5</td>
<td>3.3</td>
</tr>
<tr>
<td>LADH</td>
<td>Primary</td>
<td>6.6</td>
<td>6.9</td>
<td>7 to 8</td>
</tr>
<tr>
<td></td>
<td>Secondary</td>
<td>1.08</td>
<td>1.27</td>
<td>1.36</td>
</tr>
<tr>
<td>MADDH</td>
<td>Primary</td>
<td>5.9</td>
<td>18</td>
<td>17</td>
</tr>
</tbody>
</table>

$^a$Truhlar [101].

Table 6. Deuterium KIEs for the gas phase reaction between RCl and ClO$^-$ at room temperature (Villano et al. [117])

<table>
<thead>
<tr>
<th>RCI</th>
<th>KIE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH$_3$Cl/CD$_3$Cl</td>
<td>0.85</td>
</tr>
<tr>
<td>C$_2$H$_5$Cl/C$_2$D$_5$Cl</td>
<td>0.99</td>
</tr>
<tr>
<td>i-C$_3$H$_7$Cl/i-C$_3$D$_7$Cl</td>
<td>1.70</td>
</tr>
<tr>
<td>t-C$_4$H$_9$Cl/t-C$_4$D$_9$Cl</td>
<td>2.31</td>
</tr>
</tbody>
</table>
Complications due to solvent interaction with the reactant and transition states are thus avoided, but it is interesting to note that the IEs in Table 6 are about the same as those in aqueous solvent. The reaction of ClO\(^-\) with methyl chloride can only proceed via SN2, while that with t-butyl chloride proceeds via E2 (since SN2 attack on the Cl substituted carbon is blocked). The isotope effects for the intermediate compounds thus measure the relative contributions of the two paths; E2 becomes the dominant channel with increasing steric hindrance.

Figure 6 compares experimental and calculated 2° deuterium IEs for the Diels-Alder pericyclic addition reaction between 2-methyl butadiene and cyanoethylene (top) and 1,1-dicyanoethylene (bottom). TST calculations were carried out for potential energy surfaces corresponding to stepwise and concerted transition states. The agreement between experiment and calculation for the concerted case is excellent and clearly establishes the mechanism as concerted.

Solvent isotope effects

It is reasonable to expect that isotope substitution on solvent molecules will affect both equilibrium and rate constants. Most solvent IE studies have been carried out in aqueous media because deuterated (heavy) water is generally available at an affordable price, but also because almost all reactions of biochemical interest occur in water solution. The isotope effect on the acid dissociation constants of H\(_2\)O and D\(_2\)O has been carefully measured by Paabo and Bates [81]), \(2\text{H}_2\text{O} = \text{H}_2\text{O}^+ + \text{OH}^-\) and \(2\text{D}_2\text{O} = \text{D}_2\text{O}^+ + \text{OD}^-\)

\[
\Delta pK_a = -\log[(\text{D}_2\text{O}^+)(\text{OD}^-)] - (-\log[(\text{H}_2\text{O}^+)(\text{OH}^-)]) = 0.958
\]

Thus the acid dissociation constant of ordinary water is about ten times larger than that of heavy water. This is a significant isotope effect and has important consequences. On the other hand the acid dissociation constant of numerous weak organic acids scatter widely, \(<\Delta pK_a> = 0.5 \pm 0.1\); on average organic acids dissolved in D\(_2\)O are only about 30% as dissociated as their sister compounds dissolved in H\(_2\)O.

Many rate constants in aqueous solutions are pH or pD sensitive. Enzyme catalyzed reactions often show maxima in plots of pH (or pD) vs. rate (bell shaped curves). Consequently it is important to compare isotopic rate effects at equivalent positions on the curves (equivalent acidities). Also both equilibrium and rate constants are expected to vary in a nonlinear fashion with solvent atom fraction D (Fig. 7). For solvent KIEs one can use such nonlinearities to deduce information about the number of protons actively participating in the transition state (Quinn [82]).

Isotope effects on enzyme catalyzed reactions

One of the most active areas of isotope science in recent years has been the application of KIE theory and experiment to the elucidation of reaction paths and transition state and active site structures of enzyme catalyzed reactions (Cook [22], Cook and Cleland [23], Cleland, O’Leary and Northrup [19], Truhlar [101]).

![Fig. 7. The IE on acid dissociation constants of benzoic acid C\(_6\)H\(_5\)COOL (L = H or D) vs. atom fraction D of the solvent (Lowe and Smith [70]). The points are experimental. The bottom line is theoretically calculated, the upper line shows the linear approximation.](image-url)
condensed phase isotope effects

In this section we discuss IEs on physical properties such as vapor pressure, molar volume, compressibility, heats of vaporization, freezing points, and so forth, for liquids, solids and solutions. Some properties like the vapor pressure isotope effect, VPIE, for example, are of great practical interest because they form the basis for isotope separation processes. Of equal interest is the fact that condensed matter isotope effects are closely related to the intermolecular forces which cause condensation and therefore serve as useful probes to test ideas about the nature of condensation and the motions of molecules in condensed phases. Earlier reviews include those of Wolfsberg et al. [126], Van Hook [113], Jancso and Van Hook [58], Rabinovich [83], Jancso, Rebelo and Van Hook [56, 57], and Bigeleisen, Lee and Mandel [14] among others.

The vapor pressure isotope effect, VPIE

The vapor pressure of liquids and solutions is related to the free energy of the condensed phase in a simple fashion. The VPIE is obtained by equating partial molar free energies in condensed and vapor phases remembering \[ \mu'(c) - \mu(c) = \mu'(v) - \mu(v) \]. This leads straightforwardly (Bigeleisen [11]) to

\[
(23) \quad \ln(P'/P) = VPIE = \ln(f_f) - (B'P' - BP) + (P'V' - PV)/RT = \ln(Q'_c/Q_c/Q'_v/Q_v) - (B'P' - BP) + (P'V' - PV)/RT.
\]

The equation applies to comparisons between separated isotopes. The prime refers to the lighter isotope and the Qs are properly averaged partition functions for the vapor and condensed phases. The last two terms on the right hand side are corrections for nonideality in the vapor phase, \( B' \) and \( B \) are vapor phase second virial coefficients in the pressure expansion, \( PV/(RT) = 1 + B + \ldots \), and for isotopic differences in the condensed phase molar volumes, respectively. At low enough temperatures, say at or below the normal boiling point, \( \sim 0.7T_{critical} \), the last two terms are small and to good approximation the VPIE defines the condensed phase reduced partition function ratio, \( VPIE = \ln(f_f) \).

When considering the liquid-vapor equilibrium of a mixture of isotopes one normally defines fractionation factors \( \alpha \) and \( \alpha'' \)

\[
(24) \quad \alpha = (y'/y)(x/x') \quad \text{and} \quad \alpha'' = 1/\alpha = (y'/y)(x''/x)
\]

Here \( x \) and \( y \) denote mole fractions in the liquid and vapor respectively. To excellent approximation we obtain

\[
(25) \quad \ln \alpha'' = \ln(f_f/f'_f) + \ln(\gamma'/\gamma) = \ln(Q'_c/Q_c/Q'_v/Q_v) + \ln(\gamma'/\gamma)
\]

In the last term \( \gamma \) and \( \gamma' \) are activity coefficients in the condensed phase. Fractionation experiments are often carried out with the rare isotope at high dilution. In that case the other isotope, say for example the unprimed one, is ideal in the Raoult’s law sense, \( \gamma = 1 \), and the result is particularly simple

\[
(26) \quad \ln \alpha'' = \ln(f_f/f'_f) + \ln(\gamma') = \ln(Q'_c/Q_c/Q'_v/Q_v) + \ln(\gamma')
\]

Note that \( RT\ln \gamma' \) is the free energy of transfer of the primed isotope from its pure liquid (i.e. its Raoult’s law standard state) to infinite dilution in the unprimed solvent (the Henry’s law standard state). Very often this last term is small enough to be ignored.

In order to implement Eqs. (25) and (26) Stern, Van Hook and Wolfsberg [95] successfully applied the harmonic oscillator rigid rotor model to both condensed and vapor phases for the deuterated ethylenes. That approach has since been widely used. Van Hook [109] extended the method, improving agreement with the temperature dependence of the VPIE by treating the condensed phase external frequencies using pseudo-harmonic Gruneisen lattice theory.

The AB approximation

It is useful to have an approximate relation for VPIE, especially when complete vibrational analysis is impossible. The AB approximation serves that purpose and sometimes gives more physical insight than do detailed but very complicated calculations using Eqs. (25) and (26).

\[
(27) \quad \ln(f_f/f'_f) = A/T^2 + B/T
\]

The approximation is based on the fact that ordinarily condensed phase vibrations fall into a high frequency \( B \) group, most often the internal modes where \( \nu_i = h\nu_i/(kT) \gg 1 \) and the zero point (low temperature) approximation applies,

\[
(28) \quad \ln(f_f/f'_f) = A/T^2 = (1/2)(h\nu_i/(kT)) \sum (\nu_{i}^2 - \nu_{i}^{-2}) - (\nu_{g}^2 - \nu_{y}^2)_{high_freq}
\]

plus an \( A \) group containing the low frequencies, \( \nu_i < 1 \), treated in the high temperature approximation

\[
(29) \quad A = (1/24)(h\nu_i/k) \sum (\nu_{i}^2 - \nu_{i}^{-2}) - (\nu_{g}^2 - \nu_{y}^2)_{low_freq}
\]

A is generally positive (necessarily positive if it contains contributions only from lattice modes) and predicts \( P' > P \). B results from the shifts in internal force constants on condensation. An increased force constant on condensation (blue shift in the associated frequency) leads in the direction of an increased VPIE, a decreased force constant (red shift) leads in the direction of a inverse VPIE \( (P < P) \). The \( 1/T^2 \) dependence of the \( A \) term in Eq. (27) means that at low
enough temperature $A/T^2$ must predominate. The IE will be normal and proportional to $1/T^2$. At intermediate temperature the $B$ term, which can be positive, but which more typically is negative, may dominate. This accounts for the widely observed crossover to inverse isotope effects (especially common for the deuterated hydrocarbons). At even higher temperatures both terms decay to zero. Thus, the temperature dependence of VPIE can be complicated.

### Virial coefficient and vapor pressure isotope effects compared

Van Hook, Rebello, and Wolfsberg [114] have rationalized the isotope effects on second virial coefficients in the gas phase (VCIE) analogously to the treatment above for VPIE. It is interesting to compare $A$ and $B$ parameters of fit to VCIE and VPIE for some deuterated molecules (Table 7).

### Representative VPIEs, especially H/D effects

Solvent dependence, chromatographic effects

Figure 8 shows VPIEs per D for a few organic compounds and water and ammonia. The effects range from 10% per D normal for H$_2$O/HOD at the freezing point of H$_2$O, to 1.6% inverse for methyl acetylene at 160 K. The temperature dependences also vary widely. D for H substitution for hydrogen bonded atoms results in large normal VPIEs with large negative temperature coefficients. This is a consequence of very large blue shifts in the hydrogen bonded lattice modes on condensation which significantly outweigh the partially compensatory red shift in internal modes. For example for water there is a net red shift of $\sim$ 280 cm$^{-1}$ for the internal OH stretching modes but that is more than compensated by the appearance of three blue shifted librational modes (each in excess of 500 cm$^{-1}$) and three hindered translations. D for H substitution at nonpolar groups like methyl or methylene is different. It is characterized by red shifts in the internal stretching modes caused by the dispersion forces operative in the condensed phase which, although small outweigh the even smaller contributions of the hindered lattice modes. As a consequence H/D VPIEs for nonbonded hydrocarbons are on the order of 1 percent or so inverse per D substitution.

Since VPIE is determined by condensed phase intermolecular forces significant changes should occur when one varies the condensed phase environment. This was nicely illustrated by Dutta-Choudhury, Miljevic and Van Hook [28] who compared the VPIEs for H$_2$O/D$_2$O for the neat liquids (13.1% at 306 K) with those at infinite dilution in benzene (C$_6$H$_6$) (–1.4%). The difference is consequent to large changes in both internal and lattice frequencies on transfer from the hydrogen bonded to the nonpolar medium. The VPIE at infinite dilution corresponds to the IE on Henry’s Law constants. For the benzenes the change in VPIE on transfer of C$_6$H$_6$ or C$_6$D$_6$ from the neat liquids (–2.7%) to infinite dilution in H$_2$O or D$_2$O (4.0%) also reflects significant frequency changes. Wolff and Hoepfner [124] studied the concentration dependence of the VPIE for C$_2$H$_5$NH$_2$/C$_2$H$_5$ND$_2$ dissolved in n-butane. The IEs fall smoothly from large normal effects with large temperature coefficients at $x$(amine) = 1 to modest inverse effects at $x$(amine) = 0 (Fig. 9). The behavior is that expected as the strong hydrogen bonds of the amines are broken on dilution with butane (Hoepfner [47]). Another way to measure isotope effects on Henry’s Law constants is by using gas or liquid chromatography and a substantial literature reporting such measurements and their interpretation exists (Wolfsberg et al. [126], Van Hook [112], Maciej and Thornton et al. [71]), but space does not permit further discussion here.

### Anharmonic corrections

It is no surprise that harmonic oscillator rigid rotor calculations of VPIEs only agree with experiment over narrow temperature ranges because the harmonic model does not account for thermal expansion of the lattice. These effects are treated empirically by intro-

Table 7. $A$ and $B$ parameters for fits to $[-\text{VCIE}]$ and VPIE for selected molecules (per deuterium substitution$^a$). RPFR = $A/T^2 + B/T$

<table>
<thead>
<tr>
<th></th>
<th>$A_{\text{VCIE}}/n_D$</th>
<th>$B_{\text{VCIE}}/n_D$</th>
<th>Range (K)</th>
<th>$A_{\text{VPIE}}/n_D$</th>
<th>$B_{\text{VPIE}}/n_D$</th>
<th>Range (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CH$_4$/CD$_4$</td>
<td>233 ± 40</td>
<td>–2.34 ± 0.21</td>
<td>110/300</td>
<td>258</td>
<td>–2.89</td>
<td>90/120</td>
</tr>
<tr>
<td>C$_2$H$_6$/C$_2$D$_6$</td>
<td>53 ± 23</td>
<td>–0.74 ± 0.08</td>
<td>210/520</td>
<td>140</td>
<td>–2.86</td>
<td>115/200</td>
</tr>
<tr>
<td>NH$_3$/ND$_3$</td>
<td>2630 ± 124</td>
<td>–5.43 ± 0.34</td>
<td>298/473</td>
<td>5320</td>
<td>–12.20</td>
<td>218/273</td>
</tr>
<tr>
<td>H$_2$O/D$_2$O</td>
<td>4277</td>
<td>–6.14</td>
<td>473/723</td>
<td>16900</td>
<td>–35.20</td>
<td>273/423</td>
</tr>
</tbody>
</table>

$n_D$ = number of D atoms substituted.

$^a$ Van Hook et al. [114].
producing volume dependent best fit lattice frequencies (Gruneisen [38], Van Hook [109]). In this, the Gruneisen or pseudoharmonic approximation, one introduces a set of constants, \( \Gamma = -\frac{\partial \ln \nu_i}{\partial \ln V} - \frac{\nu_i}{2} \frac{\partial \ln(f_i)}{\partial \ln V} \), \( \nu_i \) is the Gruneisen constant for lattice translational or rotational modes and \( f_i \) is the harmonic force constant. Also, \( \Gamma = -\frac{\partial \ln V}{\partial \ln \nu_i} - \frac{\partial \ln(f_i)}{\partial \ln V} = \left( \frac{\partial \ln f_i}{\partial \ln V} \right) / \kappa \), \( \kappa = \frac{-\partial \ln V/\partial P}{\partial \ln(f_i)/\partial P} \). This relation is useful because there is much more experimental information available on the pressure dependence of vibrational frequencies than on their volume dependence. For lattice translational modes Gruneisen suggested \( \Gamma = \alpha V/\kappa \). Here \( \alpha \) is the coefficient of thermal expansion, \( \alpha = \partial \ln V/\partial T \). Typically \( \Gamma \) is about 2. For rotation/libration modes and internal vibrations, however, \( \Gamma \) is significantly different from 2, for internal modes very much different (much smaller). In the pseudoharmonic formalism the temperature dependence of the various normal mode frequencies arise only indirectly, i.e. through the temperature dependence of the volume. Anharmonic corrections for lattice modes can be important. For example in the ethylene/deuteroethylene system (Bigeleisen et al. [13]) it was found the lattice translational and rotational force constants decreased ~ 15 and 40% respectively between 104 and 180 K. One finds that changes in the volume dependent pseudoharmonic internal force constants are relatively much smaller than the changes in lattice force constants (the internal modes are stiffer and much less sensitive to intermolecular coupling). For ethylene/deuteroethylene the CH stretching constant decreases only 0.15% between 104 and 180 K, bending force constants change about 2% over that temperature range.

It is interesting to compare the empirical pseudoharmonic corrections to VPIE described above with theoretically based and more exact zero point anharmonic corrections for high frequency internal modes (Wolfsberg [125]). Unfortunately this is possible only for those relatively few diatomic and polyatomic isotope pairs for which complete vibrational analysis has yielded both harmonic frequencies and first-order (and very rarely second-order) anharmonic contributions to ZPE (Jancso and Van Hook [60]). Jancso, Jakli and Fetzer [55] and Jakli, Juhasz and Jancso [51] used VPIE data for the CHCl3/CDCl3 and CHBr3/CHBr3 pairs to conclude that the anharmonic constants for the CH(D) stretching mode change by about 4% on condensation (about ~4% for CHCl3, and 4.6% for CHBr3) at room temperature. Uncertainties in these estimates, \( (\delta \nu^* = -2.6 \pm 0.6 \text{ cm}^{-1})_{\text{ZPE}} \) for CHCl3, are less than the uncertainties in the spectroscopic values \( (\delta \nu^* = 1.3 \pm 2 \text{ cm}^{-1})_{\text{SPEC}} \). The results indicate intermolecular forces in liquid chloroform make the CH stretch for this molecule more nearly harmonic than in the vapor, less harmonic for bromoform. Even so we emphasize that vibrational analysis for the vast majority of polyatomics has been limited to the harmonic approximation, and RPFR for these molecules must be calculated using effective harmonic oscillator frequencies and frequency shifts on condensation. (However, it is now possible to use standard quantum chemistry packages to estimate anharmonic corrections to ZPE’s and RPFR’s).

The dielectric correction

A complication occurs when comparing thermodynamically observed and spectroscopically calculated RPFRs. For high intensity spectroscopic bands shape corrections for the effects of intermolecular interaction are substantial. In the spectroscopic literature they are referred to as “dielectric corrections”. For IR intense bands they can amount to 10 cm\(^{-1}\) or more and their neglect can lead to large errors in RPFR’s derived from spectroscopic measurements (Jancso and Van Hook [59], Maessen and Wolfsberg [72]).
A few examples of VPIEs

(a) Rare gases

Precise intermolecular potentials are available for monatomic He, Ne, Ar and Xe. This permits accurate calculations of the VPIE’s of these gases, their mixtures, and rare gas isotopomer mixtures. Figure 10 compares calculated RPFR’s with experiment (Lopes et al. [69]). The input parameters (well depth, ε, and size parameter, σ) for each species are completely defined in terms of the observed critical properties for each of the fluids. The agreement with experiment is remarkably good, especially considering that the VPIE’s span several orders of magnitude. Although the LJ potential is only approximate, it turns out to be an excellent choice to represent IE’s in rare gas systems. The thermodynamic properties are described with a single master (reduced) equation. Theory and experiment are in satisfactory agreement.

(b) Ethylene isotopomers

Vapor pressures of C2DH3, cis-, trans-, and gem-C2D2H2, C2D3H and C2D4, and 12C13CH4, have been measured at high precision and theoretically interpreted (Bigeleisen et al. [13]). VPIE’s for the liquids are inverse, but there is a large positive discontinuity on freezing. Ethylene was one of the first systems subjected to detailed vibrational analysis. Agreement with experiment is excellent (Fig. 11). A single isotope independent force field suffices to fit the experimental data for all seven isotopomers over a wide temperature range. The differences in the VPIE’s of the equivalent isotopomers cis-, trans-, and gem-C2D2H2 are slightly different, but superposed on that effect is an additional ZPE contribution caused by coupling between the hindered rotational modes in the liquid and certain internal vibrations.

(c) Benzene isotope effects

VPIE’s for C6H6, ortho-, meta-, and para-C6H4D2 and C6D6 are available between the normal melting and boiling points, as are data on the excess free energy of mixing (small and positive) for C6H6/C6D6, the 13C/12C effect at the boiling temperature, VPIE’s and MVIE’s (molar volume isotope effects) for C6H6/C6D6 across the entire LV coexistence range, triple to critical points, and

Fig. 11. Plot of \( T^2 \ln(f_c/f_g) = T^2 \ln(f_c/f_v) \) for isotopically substituted ethylenes. The lines are calculated from an isotope independent force field (Bigeleisen et al. [13]).
isothermal compressibility IE's between 288 and 313 K (Jakli et al. [52], Kooner and Van Hook [66], Matsuo and Van Hook [74]). HOCM calculations show good agreement with spectroscopic information and reproduce the observed subtle deviations from the law of the mean in the series D1, D2,..., Dn and the VPIE's between equivalent isomers ortho-, meta-, and para-C6H4D2 (Jancso and Van Hook [61]). With \( df_{\text{pie}}/dV \) obtained from VPIE of one isotopomer pair, and using the experimentally observed molar volume isotope effect, one calculates \( \mu^d = 1.8 \text{ J/mol} \) for the excess free energy of isotopomer mixing in the low temperature fluid in good agreement with experiment, \( 2.1 \pm 0.3 \text{ J/mol} \). The agreement furnishes strong support for the theoretical development correlating molar volume isotope effects and excess free energies of solutions of isotopomers (following section).

### Excess free energies in isotopomer solutions

Jancso, Rebelo and Van Hook [56, 57] have reviewed vapor pressure data on solutions of mixtures of isotopes \( (^{36}\text{Ar}/^{36}\text{Ar}, \text{HCl/DCl}, \text{C}_6\text{H}_6/\text{C}_6\text{D}_6, \text{poly styrene-h/d}, \text{etc.}) \) which definitively show small but significant deviations from the behavior of ideal solutions (i.e. show excess thermodynamic properties of solution). An excess thermodynamic property is just the difference between the value of that property for the real (nonideal) solution and the corresponding ideal solution. The pseudoharmonic cell model leads straightforwardly to the idea that the excess free energies of isotopomer mixtures correlate with the isotope effect on the molar volumes of the pure isotopomers (MVIE's). One calculates the free energy of mixing in a two step process: (1) compress or dilate each separated component to the molar volume of the solution, and (2) mix at constant volume, assuming the free energy change in step (2) is zero. In terms of the harmonic oscillator cell model the change in the Helmholtz free energy, \( A \), in the first step is

\[
-RT \ln(\gamma) = -\int [\text{d}A/dV] dV = -RT \sum_i [\text{d}(\ln(Q_i)/dV)\text{d}V] \ln(\nu_i)_{\text{freq}}
\]

The integral is from \( V \) to \( V' \). With Eq. (30) the contributions of internal modes are evaluated. Also apparent differences between RPFR obtained via VPIE or LVFF can be successfully rationalized (see Eqs. (23) and (25)), and the excess free energies in concentrated solutions of isotopomers, one in the other, interpreted. Examples are given in Table 8.

### Table 8. Excess free energies of some isotopomer solutions

<table>
<thead>
<tr>
<th>System</th>
<th>( T ) (K)</th>
<th>( x' )</th>
<th>( 10^4 \ln \gamma^* )</th>
<th>( G^\mu/(\text{J/mol}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(^{36}\text{Ar}/^{36}\text{Ar} )</td>
<td>84</td>
<td>0</td>
<td>-3</td>
<td>0.015</td>
</tr>
<tr>
<td>\text{CH}_4/\text{CD}_4</td>
<td>90</td>
<td>0.5</td>
<td></td>
<td>0.57</td>
</tr>
<tr>
<td>\text{HCl/DCl}</td>
<td>100</td>
<td>0.5</td>
<td></td>
<td>0.66</td>
</tr>
<tr>
<td>\text{H}_2\text{S/DD}</td>
<td>170</td>
<td>0.5</td>
<td></td>
<td>-0.92</td>
</tr>
<tr>
<td>\text{C}_6\text{H}_6/\text{C}_6\text{D}_6</td>
<td>298</td>
<td>0.5</td>
<td></td>
<td>0.58</td>
</tr>
<tr>
<td>c-\text{C}<em>6\text{H}</em>{12}/c-\text{C}<em>6\text{D}</em>{12}</td>
<td>298</td>
<td>0.5</td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>Polybutadiene-h/-d</td>
<td>296</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^{1}\) Activity coefficient of the more dilute species.

### Phase separation of isotopomer solutions

Phase separation is a straightforward consequence of solution nonideality. In regular solution theory the total excess free energy is expressed, \( G^\mu = x \mu^\mu + \mu^\mu + x \mu^\mu \). In first approximation the Flory-Huggins parameter \( \chi \) is independent of temperature and concentration, and the thermodynamic conditions for the initial liquid-liquid phase separation from symmetrical mixtures are \( x = x' = 0.5 \) and \( \gamma/RT = 2 \). For \( \gamma/RT > 2 \) there is incomplete mixing, i.e. phase separation. Typically, IE's on the excess free energy, now expressed in terms of the parameter \( \chi \), are inversely proportional to the temperature raised to some power, and directly proportional to the total isotope and phase frequency shift, \( \Delta \nu \).

It follows that liquid-liquid demixing of isotopomers will be enhanced at very low temperature, or, should one be restricted to higher temperature because the solution freezes, to molecules with very many (isotopically substituted) oscillators. The first case is realized for \(^{3}\text{He}^{4}\text{He}^{3}\text{He}^{4}\text{H}_2/\text{D}_2 \) mixtures, the second by polymer/polymer mixtures. It is long known that \(^{4}\text{He}^{2}\text{He} \) liquid mixtures phase separate at temperature below 0.9 K. The theoretical explanation, first advanced by Prigogine, has been outlined above. Similarly, mixtures of solid \(^{4}\text{He}^{2}\text{He} \) (formed at elevated pressure) and mixtures of solid \( \text{H}_2/\text{D}_2 \) phase separate, but liquid mixtures of \( \text{H}_2 \) and \( \text{D}_2 \) do not, although they do show appreciable nonideality. No other “small molecule” isotopomer mixtures phase separate. In polymers, however, while the excess free energy per oscillator is small, the effects are cumulative, and as the number of isotopically substituted bonds increases the excess free energy becomes large enough to cause phase separation in binary mixtures of perproto and perdeutero polymers. This occurs at the critical polymerization number \( N_{\text{critical}} \) where \( \gamma/RT = 2 \). Phase separation has been observed in polystyrene(\( \text{H}/\text{D} \)), polybutadiene(\( \text{H}/\text{D} \)), and polyethylene-polypropylene(\( \text{H}/\text{D} \)) mixtures. For polybutadiene the number of CH/CD substituted bonds per monomer unit is 6, and the critical polymerization number for H/D demixing is \( 1.2 \times 10^3 \) monomer units (Singh and Van Hook [93], Bates and Withnall [7]).

### Demixing of small molecule and polymer-solvent solutions

Both polymer-solvent and small molecule demixing diagrams may show upper and lower consolute branches which are functions of temperature, chain length, con-
centration, pressure, and H/D substitution (Imre and Van Hook [49]). The two branches may approach each other as some other variable (polymerization number, isotope fraction, or pressure, for example increases) and can join at a double critical (hypercritical) point. Detailed investigation of demixing as a function of these variables has been reported by groups in Tennessee, Warsaw, Budapest and Lisbon (see for e.g. Van Hook [111], Imre and Van Hook [49], Visak, Rebelo and Szydlowski [118]). Isotope effects in these systems are large. For example, in acetone-\(h\) the hypercritical MW for polystyrene is \(\sim 22,000\) at \(P = 0.1\) MPa, but in acetone-\(d\) the hypercritical MW has dropped to 13,500 (i.e. \(\ln[\text{MW}_{\text{d},-\text{d}}/\text{MW}_{\text{d},+\text{d}}] = 0.49\) which is a remarkably large IE; deuteration operates in the same direction as a drop in pressure, i.e. a decrease in solvent quality). At constant MW and \(P = 0.1\) MPa the solvent IEs are also large, for example \(\ln(T_{\text{UCST}'}/T_{\text{UCST}})_{\text{13,500}} \sim -0.22\) and \(\ln(T_{\text{UCST}'}/T_{\text{UCST}})_{\text{7500}} \sim -0.09\) (UCST = upper critical solution temperature). The effect of hypercritical enhancement is obvious.

Another interesting example: critical demixing loci of small molecule solutions of 3-methylpyridine (3-MP) in H\(_2\)O/D\(_2\)O solvents are shown in \(T, P, x(3\text{-MP})\) space, Fig. 12 (Visak et al. [118]). At atmospheric pressure a 70 K closed immiscibility loop, \((T_{\text{UCST}} - T_{\text{LCST}})\), is observed for (3-methylpyridine + D\(_2\)O) and the immiscibility gap is modestly pressure dependent. In the \((p,T)\) projection the phase diagram shows a characteristic “hour-glass” shape. On addition of H\(_2\)O the gap shrinks, becomes more and more pinched at the waist, and at 21 wt.% D\(_2\)O it disappears completely as the phase diagram changes from this “hour-glass” to a shape that resembles the UCST/LCST configuration in the \((T,x)\) plane – two immiscible domes, an upper and a lower. With continued addition of H\(_2\)O, holding (3-methylpyridine + water) at the critical concentration, the upper and lower immiscible branches move further and further apart, until finally at high enough H\(_2\)O/D\(_2\)O (17 wt.% D\(_2\)O) the low-pressure branch is no longer present at atmospheric pressure, dropping below the \(P = 0\) isobar. For (3-methylpyridine + H\(_2\)O) the miscibility gap between the upper and lower branches amounts to \(\sim 160\) MPa. The phenomenon corresponds to an impressive pressure shift of hundreds of atmospheres merely upon (H/D) solvent isotope substitution. Very large isotope effects like these seem to be limited to the hypercritical regions of phase diagrams, i.e. not too far from thermodynamic divergences of the type \((dP/dT)_c = \infty\) or \((dT/dP)_c = \infty\).

**Other solubility IE studies**

Scharlin and Battino [89, 90] have reviewed gas solubility IE’s in H\(_2\)O and D\(_2\)O. Rabinovich [83] reports solubilities of H\(_2\) and D\(_2\) in various solvents, and has also compiled solvent IE data (H\(_2\)O/D\(_2\)O) for numerous ionic electrolyte solutions.

**Isotope effects on some other molecular properties**

**Dipole moments, polarizabilities**

Rabinovich [83] and Wolfsberg et al. [126] have reviewed data on dipole moment and polarizability isotope effects. The effects are small. Polarizability IEs are most conveniently measured using differential refrac-

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![Fig. 12. Two representations of the phase diagram for 3-methylpyridine/water(H/D). (a) \(T-P-x(3\text{-MP})\) for three different H\(_2\)O/D\(_2\)O concentration ratios. The inner ellipse and corresponding critical curves hold for \((0 < W(\text{D}_2\text{O})/\text{wt.} % < 17)\). The intermediate ellipse is for \((17(\text{D}_2\text{O})/\text{wt.} % < 21)\), and the outer \((21(\text{D}_2\text{O})/\text{wt.} % < 100)\); (b) Phase diagram at approximately constant critical concentration 3-MP \((x \sim 0.08)\) showing the evolution of the diagram as the deuterium content of the solvent varies (Visak et al. [118]).](image-url)
tometry and Urbanczyk and Van Hook [102, 103] and Wieczorek, Urbanczyk and Van Hook [121] describe a continuous dilution differential refractometric technique to measure isotopic refractive index differences, refractive indices, and excess volumes for solutions of isotopomers. The dipole moment and polarizability express the first and second order interactions of a molecule with an impressed electric field. Isotope effects on these electronic properties are a consequence of isotopic differences in vibrational anharmonicity. Van Hook and Wolfsberg [116] have discussed the theoretical origin of the effects and correlated polarizability and molar volume IEs. Many years ago Bell and Coop [8] advanced the idea that the IE on dipole moment, $\Delta \mu$, for diatomic molecules is given by the product of the difference averaged bond lengths and the dipole moment derivative. In the diatomic case, (HCl/DCI for example), one obtains

$$\Delta \mu = \mu_{H} - \mu_{D} = (\partial \mu/\partial r) (\langle r_{H} \rangle - \langle r_{D} \rangle)$$

It is well established that the average lengths of CH bonds are consistently 0.003 to 0.004 Å longer than the corresponding CD bonds in the ground vibrational state due to anharmonic effects. The dipole moment derivative, $(\partial \mu/\partial r)$, at the equilibrium bond length is available from theoretical calculation or spectroscopic measurement (via precise measurements of IR intensities of vibration-rotation bands). Calculations based on Eq. (31) yield predicted dipole moment IE’s in reasonable agreement with experiment.

Isotope effects on NMR shielding

An enormous literature dealing with isotope effects on NMR parameters has accumulated over the years including studies on chemical shift and coupling constant IE’s, the use of isotope labeling for molecular structure determinations, the study of fast exchange processes, and the use of NMR as an analytical tool to measure isotope fractionation (Diehl et al. [27], Hansen [39]). Our attention is limited to IE’s on chemical shifts (nuclear shielding) due to isotopic differences in averaging of rotation-vibration contributions to molecular properties (rovibrational effects). The theory described by Jameson [53] points out the isotope effect on shielding ($\sigma$) depends on vibrationally averaged bond lengths and angles (different due to anharmonic effects) weighted by a set of isotope independent shielding parameters $(\partial \sigma/\partial r)$, $(\partial^2 \sigma/\partial (r, \theta))$, $(\partial \sigma/\partial \alpha)$, etc. which are most reliably obtained from $ab initio$ quantum mechanical calculation. An excellent illustration of the application of rovibrational theory to NMR shift data, including comparison with experiment is given in Table 9. The agreement is reasonable although the calculated values are consistently about 20% larger than experiment, probably because the bond length dependence of nuclear shielding, $(\partial \sigma/\partial r)$, is overestimated by the choice of Hartree-Fock wave functions used in the calculation.

### Molar volume isotope effects

Molar volume isotope effects (MVIE’s) for common liquids between the triple and boiling points amount to a few parts per thousand or less per H/D substitution. At temperatures well below the critical point liquid molar volumes generally lie within 10% or so of the values for the solids at the triple point and it is reasonable to assume a model where liquid state molecules, on average, are in contact with one another. For convenience this approach is called “the mechanical model”. At higher temperatures the expansivity and compressibility increase rapidly, the liquid expands dramatically (typically critical volumes are some 3 or 4 times molar volumes at $T_{TRP}$), and the mechanical model is inappropriate. Except for $^3$He/$^4$He MVIE data are only available for H/D substituted molecules, because the effects for heavier isotopomer pairs are too small to be conveniently measured. Both positive and negative effects are observed for low temperature liquids, but negative values (i.e. $V_H > V_D$) only occur for strongly associated (hydrogen bonded) liquids. MVIE’s are well known for (CH$_4$/CD$_4$, C$_6$H$_6$/C$_6$D$_6$, and H$_2$O/D$_2$O) between $T_{TRP}$ and $T_{Critical}$. In each case MVIE is relatively flat at low temperature (i.e. $\delta \ln (V')/\delta (T) - 0$ for ($- 0.45 < T/T_{Critical} < - 0.6$)) but at higher temperature it drops off rapidly and reaches large negative values. MVIE’s for $^3$He/$^4$He and H$_2$/D$_2$, on the other hand, are large and positive (hundreds of parts per thousand) and increase rapidly with temperature. At these higher temperatures MVIE’s are best treated with corresponding states theory (Van Hook, Rebello, Wolfsberg [115]). The discussion here is concerned with the application of the mechanical model to H/D effects at lower temperatures. That model, introduced by Bartell and Roskos [6], assumes that the major part of MVIE for polyatomic molecules at low temperature is due to the isotope effect on the XH/XD vibrational amplitudes. For example, for C$_6$H/C$_6$D, taking the IE on CH/CD vibrational amplitude as 0.005 Å, one finds (modeling benzene as a disk) $\Delta V/V = 2\Delta r/r = 2(0.005)/3.7 = 0.02$; in reasonable agreement with the observed value, 0.0021. Here 3.7 Å is the Van der Waals radius of benzene. Similarly for CH$_3$/CD$_3$, $\Delta V/V = 3\Delta r/r = 3(0.005)/(2.3) = 0.070$ (observed = 0.011). The predicted MVIE’s are in rough agreement with observation, but the model is crude and needs refinement. It is unable to predict the temperature coefficient of the MVIE’s for non-polar molecules and it is not applicable to complicated liquids, for example ones with hydrogen bonded networks like water. For

| Table 9. Calculated and observed IE’s on carbon-13 shielding constants of methane isotopomers at 300 K, -[<\sigma>] - <\sigma>] | (in ppm) |
|---------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| $<\sigma>$ | $^{13}$CH$_3$D | $^{13}$CH$_2$D$_2$ | $^{13}$CHD$_3$ | $^{13}$CD$_3$ | $^{13}$CD$_2$ |
| Calculated | 0.250 | 0.500 | 0.746 | 0.992 | 0.992 |
| Experiment | 0.187 | 0.385 | 0.579 | 0.774 | 0.774 |

*Raynes et al. [84].

*Alei and Wagman [1].
H_2O/D_2O the MVIE is inverse and shows a complicated temperature dependence connected with IE's on the amplitudes of the external librational modes and the gradual loss of the hydrogen bonded structure with increasing temperature. The librational motions occur about the center of mass which is isotope dependent for these non-centrosymmetric molecules and this accounts for the anomalous behavior (Dutta-Choudhury and Van Hook [29], Wolfsberg et al. [126]).

**Corresponding states and IE's: critical property IE's**

In spite of the success of corresponding states (CS) theory in describing the PVT properties of fluids, it has been rarely applied to the interpretation of IE's. In CS the temperature dependence of the molar volume and the conditions for liquid-vapor coexistence are expressed using the critical properties of the fluid and one or more additional parameters. In applying CS to isotope effects, quantization, essential to the understanding of thermodynamic IE's, is introduced in terms of the critical property IE's. CS ignores the subtleties of molecular structure and vibrational properties and therefore cannot be expected to be as useful for rationalization of vapor pressure IE's as it is for molar volume IE's. The VPIE depends on subtle isotopic differences in the vibrational properties of the coexisting vapor and liquid phases. The molar density IE, on the other hand, is a much simpler function of the molecular structure and overall motion in the condensed phase. Corresponding states fits to vapor pressure IE's are not as good as those to molar volume IE's, even after the empirical introduction of an additional isotope dependent parameter. Van Hook, Rebelo, and Wolfsberg [115] recently described a modified Van der Waals corresponding states equation which yielded useful representations of VPIEs and MVIEs over the complete liquid range, T_{Tr} to T_{critical}. They also developed correlations of critical property IE's with each other and near critical values of the VPIE (Fig. 13). This permits the estimation of the IE's, ΔT_{critical}/T_{critical}, ΔV_{critical}/V_{critical}, and ΔP_{critical}/P_{critical} from the critical temperature of the more common isotopomer, T_{critical}', and a near critical value of VPIE. That information in hand, the corresponding states equation yields ln(V'/V) across the entire liquid coexistence range. This is a very useful result: few data are available for MVIEs except near room temperature, and experimental programs to expand the data base are difficult, expensive, and tedious to implement. Corresponding states analysis offers an attractive alternative.

**IEs in geochemical and environmental studies**

Measurements of the isotope ratios of naturally occurring materials have long provided important information in geochemistry, meteorology, evolutionary biology, and archeology. For example, comparisons of isotope distributions in coexisting minerals, with those measured in the laboratory, or with theoretically calculated ratios, have been used to specify the conditions of formation of the mineral. Similarly, fractionation of the hydrogen and oxygen isotopes of

![Fig. 13. The correlation of ln(T_{c}'/T_{c}) with ln(P'/P) at T_{c} = T_{c}/T_{c} = 0.975 (Van Hook et al. [115]).](image-url)
water during natural phase changes (i.e. evaporation, rain, snow, or ice formation), or chemical changes (precipitation of hydrated minerals, biological/metabolic equilibration, etc.), furnish information on the nature of those processes. Such information can be difficult to obtain because one is interested in small changes, say a percent or so, in the natural abundance ratio which is typically small itself. For example the oxygen isotope ratio in natural waters, $R = ^{18}O/^ {16}O$, normally lies in the range $(0.0019 < R < 0.0021)$. Thus a $1\% ^{18}O/^ {16}O$ IE characterizing some physical or chemical change of interest amounts to a change of about $\Delta R = 0.00020 \times 1.01 - 0.00020 = 0.00002$, i.e. 20 ppm (parts per million). For $D/H$, $R = 1.58 \times 10^{-4}$ and a $1\%$ effect amounts to only $\Delta R = 1.6$ ppm. These are small changes indeed and careful attention to experimental detail and sophisticated instrumentation are required to assure sufficient precision and accuracy to distinguish between alternate explanations for observed fractionations. High precision isotope ratio mass spectrometry is the technique of choice for measurements of geochemical and environmental IEs. Criss [25], Galimov [33], Hoefs [46], and Richardson and McSween [85] afford entry points into the literature.

Unfortunately space permits only a cursory glance at several examples of geochemical interest.

Geochemical temperature scales

In the late 1940’s H. C. Urey recognized that the temperature dependence of the isotope exchange equilibrium between water and calcite (the principal mineral in marine limestones) could be employed as a “paleo-thermometer. At 298.15 K the fractionation factor for calcite-water is 1.0286,

$$\text{(32)} \quad \text{CaC}^{16}\text{O}_3 + \text{H}_2\text{O} \leftrightarrow \text{CaC}^{16}\text{O}_2\text{O}_3 + \text{H}_2\text{C}^{16}\text{O}_2; \quad \alpha = 1.0286$$

which is to say $\delta^{18}O = 1000(\alpha - 1) = +28.6$, provided the calcite was laid down from standard ocean water. Since the temperature dependence of $\alpha$ was known ($da/dT \sim -0.23 o/ooo K^{-1}$), accurate $\delta^{18}O$ measurements on ancient marine sediments could be used to determine water temperature at the time the sediment was precipitated. Details were reported in 1951 (Urey et al. [105]). In addition to presupposing development of instrumentation of the necessary precision, the method depended on two important assumptions: (1) the isotope concentration of the ocean has remained constant over time and position; (2) the mineral was deposited in thermodynamic equilibrium with ocean water, and has remained unchanged over long periods of time. The carbonate paleo-temperature scale has aided our understanding of the climate changes Earth has experienced during the last $\sim 3$ million years, a period of intermittent glaciation. A prime criticism of the $\delta^{18}O$ carbonate paleo-temperature scale is the absence of any independent measure of the isotopic composition in ancient seawater. Therefore isotopic fluctuations in the sediments do not necessarily correlate with changes in temperature alone. Even so, the basic idea remains valid.

Many ore deposits contain coexisting mineral pairs which have been formed in equilibrium as judged by independent (non-isotopic) criteria. In such cases the mineral-mineral fractionation values for a given M-N mineral pair, $\Delta = \delta_N - \delta_M$, correlate straightforwardly with the geochemical temperature of mineral formation (it being supposed that the equilibrium is “locked-in” as temperature rapidly falls after the initial ore formation). At the high temperatures of formation of metamorphic or igneous rock deposits the isotope fractionation for many mineral systems closely follows the $1/T$ dependence predicted by the high temperature approximation to equilibrium isotope fractionation.

Isotope hydrology

Large changes in the isotope signature of water are associated with condensation, evaporation, or melting. At equilibrium the fractionation between two coexisting phases is a function of the temperature. When water evaporates from the ocean it shows $\delta D = -80 o/o$ and $\delta^{18}O = -9 o/o$, less (i.e. more negative) at high latitudes, more (less negative) at lower latitude. No matter the location, however, atmospheric water vapor always has negative $\delta D$ and $\delta^{18}O$ relative to standard mean ocean water (SMOW) because the vapor pressure of H2$^{18}$O is greater than those of HD$^{18}O$ or H2$^{16}$O at all temperatures of meteorological interest. Fractionation data are usually displayed using a three isotope plot (Fig. 14). Meteoric waters lie on or close to the correlation line $\delta D = 8.2 \delta^{18}O + 10.4$. Shallow ground-waters follow the same pattern, but waters from deep aquifers, geothermal waters, ice cores, etc. can and do lie well off the line.

Ice cores

The ice sheets of Greenland and Antarctica are several kilometers thick and preserve a sequential record of high latitude precipitation over tens of thousands of years. This record has been studied by the deep core Arctic and Antarctic drilling programs during the past thirty or more years. The "Vostok" core drilled in the high Antarctic through ~ 3.7 km of ice sampled fossil ice as old as ~ 350,000 years. The Vostock data show several sharp saw-toothed climatic cycles. The ice samples

![Fig. 14. Three isotope plot showing $\delta^{18}O$ and $\delta D$ values for meteoric waters (Wolfsberg et al. [126]).](image-url)
corresponding to the maxima are lower than modern precipitation at this location by δD = −50 ‰ and δ18O = 6 ‰ suggesting that average temperatures during those periods of glaciation were about 10 K lower than present day ones. It is interesting that the pattern of the ice core data reflects that of the tropical sedimentary carbonate cores. The δ18O saw-tooth pattern occurs at the same intervals, but has smaller amplitude and is of opposite sign. The cycles represent a combination of the effects of climate change and changes in δ18O in seawater caused by the growth or retreat of the polar ice caps (Shakleton et al. [91]).

**Paleo-diets**

As a final example in this section we consider the application of geochemical isotope techniques to diet. The complex biochemical paths in organisms offer many ways to fractionate isotopes by both kinetic and equilibrium processes. It is therefore expected, and it is observed, that the different carbon, hydrogen, oxygen and nitrogen atoms in organic residues show compound and atom-site specific isotope fractionations (Galimov [33]). A thoroughly studied example that of the several different pathways of photosynthetic carbon fixation by plants. Terrestrial plants using C3 (e.g. rice, wheat, beets, potatoes, legumes) and C4 (e.g. maize, corn, sorghum, sugar cane) photosynthetic pathways show different 13C fractionations. The distinction between C3 and C4 is in the number of carbon atoms found in the products of the initial metabolic conversion step. C4 plants such as maize contain relatively large amounts of carbohydrate and protein and typically show δ13C ~ −15 ‰. This compares to the −25 ‰ found in C3 plants which contain larger amounts of lignin, cellulose and lipids. Marine plants (including plankton) show 13C/12C about 8 ‰ lower than C3 plants, most likely because they fix marine HCO3 − (0 ‰) rather than atmospheric CO2 (∼−7 ‰). Since, roughly, the degree of isotope fractionation in the plants lying at the base of the food chain is expected to carry over to the animals which consume them, it follows that such isotope ratios can be used to study animal (and human) ecology and archeology (“they are what they eat, ± 1 or 2 per mil”). That turns out to be the case. In one interesting study the arrival of maize agriculture from Central America into the lower Illinois valley around 1000 AD was demonstrated by measuring δ13C of human bone collagen as a function of time. Because of a lack of direct archeological evidence concerning ancient Native American agricultural practice the isotope data helped solve an otherwise intractable problem. This methodology has been widely applied (DeNiro [26]).

**Unimolecular processes. The “mass independent” fractionation of ozone**

A straightforward consequence of the standard theory of equilibrium isotope effects is that the ratio of ratios of fractionations can be calculated from the differences in mass and mass distribution of the reacting particles, and this is independent of the particulars of any fractionation process. Thus for a three isotope mixture, for example H, D, T or 16O, 17O, 18O, the equilibrium theory predicts the ratio of concentrations ([D]/[H])/([T]/[H]) or ([17O]/[16O])/([18O]/[16O]) to be approximately 0.5. For oxygen isotope effects that line, generally known as the terrestrial fractionation line (TFL), but occasionally as the earth-moon line, has been experimentally established by measurements on many thousands of samples of oxides, silicates, carbonates, etc. from earth and moon and is completely consistent with isotope effect theory. For the three isotope set 16O:17O:18O along TFL δ18O = m δ16O with m = (1 − 16/17)/(1 − 16/18) = 0.529. It was therefore highly surprising to discover that oxygen isotope fractionation for certain inclusions found in meteorites did not follow the expected pattern, but unexpectedly showed ([17O]/[16O])/([18O]/[16O]) ~ 1, i.e. “mass independent fractionation” (Fig. 15). Then current thinking assumed that all possible geophysical and geochemical processes produced mass dependent isotope fractionations (m ≈ 0.53 for oxygen), and large differences from that value most likely reflected the intervention of some nuclear event. It forced the conclusion that the meteorites had sampled a region where the “primordial soup” was significantly different than for the earth/moon. This conjecture, however, was shown experimentally to be unnecessary by Thiemens and coworkers in the early 1980s (Thiemens and Heidenreich [100], Thiemens [99]) who studied (nonequilibrium) isotope fractionation during the production of ozone from oxygen in an electric discharge. The product ozone was equally enriched in 17O and 18O, so m = 1, rather than the expected mass dependent value, m = ½. The slope of the mass-independent line is the same for terrestrial ozone synthesis as it is for the meteorite data. This leads to the conclusion that the mass independent slope for the meteorite fractionation might have a chemical rather than a nuclear origin (assuming some plausible mechanism which transfers ozone fractionation from the low pressure gas to Ca/Al oxide inclusions in meteorites).

![Fig. 15. Three isotope plot of inclusions in chondritic meteorites compared with the terrestrial fractionation TFL line](source)
Also, measurements on natural ozone samples collected at high altitude, far into the stratosphere, show mass independent fractionation, not only for ozone but for other oxygen containing molecules (sulfates, nitrogen oxides, and carbon dioxide). These upper atmosphere fractionations can be transferred from the low pressure gas to aerosol particle surfaces and eventually to precipitated bulk phase solids.

Figure 16 compares measured and calculated isotope fractionations for all ozone isotopomers prepared from an enriched oxygen precursor (Gao and Marcus [34]). In this figure (16O16O16O, 16O16O17O, etc. are represented as; 666, 667, 676, 677, etc.). The calculations are in quantitative agreement with experiment. The pattern shown in Fig. 16 does not vary a great deal with the temperature or pressure at which the synthesis is carried out and is consistent with experiments on natural abundance starting material, and with samples collected from the upper atmosphere. In the calculations Marcus and coworkers applied their modernized version of the RRKM (Rice-Ramsberger-Kassel-Marcus) theory of unimolecular reactions to the ozone problem. They considered a kinetic scheme in which a collisionally activated and short lived intermediate, O3†, (O3 + O → O3†) undergoes a rapid series of steps which redistribute its internal and rotational energy, but then at some point may dissociate by one of two available paths, for example 16O16O16O† → 16O16O + 16O or 16O16O16O† → 16O + 16O18O, or as a third alternative may collide with bath molecules and deactivate to stable ozone, 16O16O18O + bath → 16O16O18O + bath. The rate constants for the three processes for each activated isotopomer species are dependent on the kind and number of vibrational and rotational quantum states characterizing the activated and product species. The calculation of the overall rate of formation for each isotopomer involves integrations of these state specific reaction probabilities weighted by the density of states for both activated species and product. It is important to recognize that in no sense is an equilibrium between activated state and reactants ever established, nor is one ever approached. It is for that reason that ordinary Bigeleisen-Mayer kinetic isotope theory is not applicable to this problem, nor for that matter to unimolecular reactions in general. The calculated isotope enrichments for the different possible ozone isotopomers are compared with experiment in Fig. 16. The agreement is excellent. The nearly mass independent fractionation observed for the asymmetric isotopomers is due to “symmetry driven” isotope effects. The asymmetric ozone isotopomers contain a greater density of reactive states as compared to the symmetric. The asymmetric species can distribute energy better and is more likely to couple to the exit channel which leads to a stable molecule. For practical application of the theory it is a requirement that these state densities be known (or calculated) for all isotopomer species of interest, and at high precision. An important quality of the Marcus model is that it provides a theoretical structure for the understanding of quantum state density isotope effects in general, and is not specifically confined to the formation of ozone itself. This feature is important because we are now aware that “mass independent” fractionations occur widely in nature. The theory aids in prediction of where such effects will be likely found, and once found, in rationalizing how they were chemically produced.

References

Isotope effects in chemistry

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