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Progress in theoretical models of the ICRF mode conversion

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Abstract. The technique of heating the plasma with the electromagnetic waves in the ion cyclotron range of frequencies (ICRF) has many important applications that may lead to improved performance of tokamaks. Recently the heating efficiency of the ICRF mode conversion scenarios characterized by a narrow power deposition profiles received much attention. This paper highlights progress in the theoretical models of the ICRF mode conversion that allowed to achieve a successful experimental realisation of these scenarios in present-day tokamaks.

Key words: ICRF heating • ion-ion hybrid resonance • mode conversion • tokamak

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Introduction

Heating of the plasma with the electromagnetic radiation in the ion cyclotron range of frequencies (ICRF) is a technique widely used in the present-day fusion devices. It is also one of the four heating methods foreseen for the installation at the ITER tokamak [6]. Heating the plasma with the fast magnetosonic waves (FW) has a number of advantages compared to other radio-frequency (RF) methods: the technological feasibility of the RF complex (generators, transmission lines, antennas) for this frequency range ($f \approx 20$–120 MHz); absence of density limits for the FW to access the high-density plasma core; a satisfactory coupling efficiency; and finally the existence of various efficient linear damping mechanisms that allow us to heat either ions or electrons, depending on the chosen heating scenario, etc. [2, 13].

The fundamental ICRF heating of the single-ion species plasma is usually characterized by a low efficiency due to the unfavorable FW polarization at $\omega = \Omega_i$ [17]. To avoid screening of the left-hand polarized component of the RF electric field E_+ responsible for the ion heating one usually uses plasma containing two ion species with different charge-to-mass ratio [1]. Depending on the minority/majority density ratio, two heating regimes may be identified. An ICRF scenario that guarantees the best performance involves small minority concentrations and is known as a minority heating (MH). This regime is characterized by the generation of supra-thermal minority ions accelerated to energies in the MeV range, which leads to an indirect bulk ion or electron heating. The record 22 MW ICRF power was coupled to the JET plasma operating in the MH regime.

An increase in the minority concentration leads to the transition to the mode conversion (MC) regime. In this scenario the FW is partially converted to a short wavelength mode at the ion-ion hybrid (IIH) resonance, the location of which is determined by the minority concentration. The converted wave is commonly absorbed by electrons in a narrow spatial region due to a strong upshift of the poloidal mode number and the subsequent upshift of the parallel wave number. In contrast to the indirect bulk plasma heating observed in the MH scenario, the MC regime is characterized by a direct localized electron heating that occurs on a short timescale of the electron-electron collisions.

The MC regime has been actively studied in the last couple years due to a number of reactor-relevant applications of this scheme which go beyond the heating effect itself [15]. These include: a localized heat source for transport studies; the generation of plasma rotation and the current drive; a method of impurity control; a diagnostic tool to measure the plasma composition; the channelling of the power from alpha particles directly to ions in a burning D-T plasma, etc. With the ICRF antennas located at the low field side (LFS) – a case relevant for the present-day tokamaks – a successful experimental implementation of the MC scenario is in general more difficult than that of the MH scheme. The reason is that the mode conversion at the IIH resonance is accompanied by the reflection of the FW, which in the case of a large minority concentration makes the single-pass RF absorption very inefficient. The fraction of the converted power is sensitive to many plasma parameters and can be large only for a narrow range of values of the minority concentration.

The aim of this paper is to describe different theoretical models used for the analysis of the ICRF mode conversion. Starting with the classical Budden model, we discuss how the mismatch in perpendicular FW wavelength at the opposite sides of the mode conversion layer affects the scattering coefficients. Then we focus our attention on the possibility that the mode conversion may be enhanced as a result of interference of the reflected fast waves. In particular, we explore the cases relevant for recent heating experiments on the JET tokamak, in which an additional reflection of the FW occurred at the high-field side (HFS) low density cutoff and the supplementary mode conversion layer.

Tunneling factor evaluation

The simplest approach used to describe the propagation, absorption and mode conversion of the FW is based on the wave equation written in the slab approximation [17]:

$$\frac{d^2 E_y}{d x^2} + k^2_{\perp,\mathrm{FW}}(x) E_y = 0 \tag{1}$$

Here, the confining toroidal magnetic field is assumed to be in the 'z' direction, with 'y' and 'x' axes defining the poloidal and radial directions, respectively.

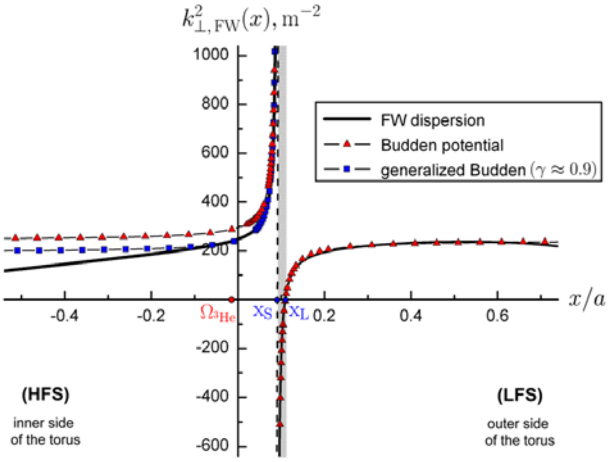

Fig. 1. A typical FW dispersion curve in a two-ion component plasma.

Plasma is assumed to be non-uniform in the radial direction due to the inhomogeneity of the toroidal magnetic field and the plasma density. The FW perpendicular wave number, $k_{\perp,\mathrm{FW}} = (\omega/c)n_{\perp,\mathrm{FW}}$ varies in the radial direction (Fig. 1) according to:

$$n^2_{\perp,\mathrm{FW}}(x) = \frac{(L - n^2_{\parallel})(R - n^2_{\parallel})}{S - n^2_{\parallel}} \tag{2}$$

where S, L and R are the components of the plasma dielectric tensor in the notation of Stix, and $n_{\parallel} = ck_{\parallel}/\omega$, where $\omega = 2\pi f$ is determined by the RF generator. The parallel wave number $k_{\parallel}$ is determined by the antenna geometry and the antenna phasing. The tensor components $S = \varepsilon_1$, $L = \varepsilon_1 + \varepsilon_2$, $R = \varepsilon_1 - \varepsilon_2$ may be approximated by the following expressions:

$$S \approx \sum_i \frac{\omega^2_{pi}}{\Omega^2_i - \omega^2}, \quad L \approx \sum_i \frac{\omega^2_{pi}}{\Omega_i(\Omega_i - \omega)}, \quad R \approx \sum_i \frac{\omega^2_{pi}}{\Omega_i(\Omega_i + \omega)} \tag{3}$$

where $\omega_{pi} = \sqrt{4\pi n_i q_i^2/m_i}$ and $\Omega_i = q_i B/m_i c$ are respectively the plasma and the cyclotron frequency of the ion species forming the plasma.

According to Eq. (2), the resonance condition for the FW can be written as $S = n^2_{\parallel}$. In the ion cyclotron range of frequencies this condition can be fulfilled if plasma consists of at least two ion species with different q_i/m_i ratio. This resonance condition defines the radial position of the mode conversion layer x_S where the FW is partially converted to a short-wavelength mode. It should be noted that the resonant absorption rate calculated within the cold-plasma model described by Eq. (1) is equal to the fraction of the FW converted to the small-scale mode calculated within a more complex full-wave model retaining finite plasma temperature [18]. As shown in Fig. 1, the IIH resonance is accompanied by an L-cutoff x_L on the LFS, defined by the condition $L = n^2_{\parallel}$. The IIH resonance and the L-cutoff together form the evanescence (mode conversion) layer which acts as a barrier for the FW propagation. Inside this region $k^2_{\perp,\mathrm{FW}} < 0$, so the FW may transfer energy through the barrier only via the tunneling effect.

The evanescence layer may be quantitatively characterized by the tunneling factor. For an isolated cutoff-resonance pair it is equal to the product of the asymptotic perpendicular FW wave number and the width of the conversion layer, $\eta = k_A\Delta$. A more general definition of the tunneling factor involves the spatial variation of the FW wave number only within the mode conversion layer [10]:

$$\eta = \frac{2}{\pi}\int_{x_S}^{x_L} \left|k_{\perp,\mathrm{FW}}(x)\right| \mathrm{d}x \tag{4}$$

For low values of $k_{||}$ one can derive an approximate formula for the tunneling factor:

$$\eta \approx 4.9\sqrt{n_e(10^{19}\,\mathrm{m}^{-3})}\,L_\mathrm{B}(\mathrm{m})\sqrt{\frac{Z_2}{A_2}}\left(1-\frac{1}{\mu}\right)^2\sqrt{1+\mu}\,Z_2X_2 \tag{5}$$

where n_e is the local electron density, $L_\mathrm{B} = R_0 + x_S$ is the characteristic scale of the magnetic field variation, $\mu = q_1m_2/q_2m_1$, and $X_2 = n_2/n_e$ is the concentration of the minority ions. For $k_{||} \neq 0$ the tunneling factor is significantly reduced, but remains to be a linear function of the minority concentration. For "standard" ICRF scenarios with "light" minority ions, e.g. (H)D, (^{3}He)D, and (D)T (the species in brackets denotes the minority ions) one obtains $\mu < 1$, while for the "inverted" scenarios with "heavy" minority, e.g. (^{3}He)H and (T)D, one obtains $\mu > 1$. As follows from Eq. (5), the tunneling factor for the "inverted" scenario is substantially larger than that for the "standard" scenario if other parameters are identical. As a result, the MC regime for the "inverted" scenario is reached at a much lower minority concentration [20]. For example, the MC regime in (^{3}He)D plasma is observed at $X(^3\mathrm{He}) \sim 15\%$, while in the (^{3}He)H plasma it is reached at $X(^3\mathrm{He}) \sim 5\%$.

The Budden model

One of the first models used to describe the propagation of the FW through the mode conversion layer was the Budden model [3]. In this approach the FW dispersion is modeled by the expression (Fig. 1):

$$k^2_{\perp,\mathrm{FW}}(x) = k^2_\mathrm{A}\left(1-\frac{\Delta}{x-x_S}\right) \tag{6}$$

For this model an analytic solution to the wave Eq. (1) may be obtained, written in terms of the confluent hypergeometric functions. The transmission coefficient does not depend on the side from which the FW approaches the barrier, and it is exponentially decreasing with increasing evanescence layer width:

$$T_\mathrm{B} = e^{-\pi\eta} \tag{7}$$

However, there is an asymmetry in the reflection and conversion coefficients with respect to the side from which the FW is incident. For the waves incident from the HFS (the FW meets the resonance first) there is no reflection, i.e. $R_\mathrm{HFS} = 0$. The fraction of the converted power is equal to $C_\mathrm{HFS} = 1 - T_\mathrm{B}$. By increasing the minority concentration to make the evanescence layer sufficiently thick one may achieve an almost total conversion.

However, due to the inner side space limitations most of the present-day tokamaks have the RF complexes located outside of the machines. The waves launched from the LFS approach the mode conversion layer from the side of the L-cutoff. In this case the Budden model predicts a non-zero reflection

$$R_\mathrm{B} = (1 - T_\mathrm{B})^2 \tag{8}$$

which dominates if the layers are sufficiently thick. The mode conversion coefficient for the LFS incidence cannot exceed 25%:

$$C_\mathrm{B} = T_\mathrm{B}(1 - T_\mathrm{B}) \tag{9}$$

This maximum is achieved for a semi-transparent layer with $T_\mathrm{B} = ½$ and $\eta \approx 0.22$.

Kaufman *et al.* explored the dissipative Budden model [8]. They showed that MC is a two-step process with the transmission occurring at the first step and reflection at the second step. They also extended the Budden model to include the dissipation of the converted wave, which leads to a reduction in the FW reflection coefficient for the LFS incidence and could be possibly used as a diagnostic test to measure the density of the alpha particles.

The generalized Budden model

If one accounts for a mismatch in the FW wavelength at the opposite sides of the conversion layer (Fig. 1), then it is also possible to derive analytic expressions for the scattering coefficients [9, 19]. The mismatch in the perpendicular wavelength is characterized by the parameter $\gamma = \lambda_\mathrm{LFS}/\lambda_\mathrm{HFS}$. One can show that the transmission coefficient remains symmetric, although it has a somewhat lower value for $\gamma \neq 1$:

$$T(\eta,\gamma) \approx T_\mathrm{B}(\eta) - \delta T(\eta)(\gamma - 1)^2 \tag{10}$$

Within the generalized Budden model a non-zero reflection from the barrier appears for the waves incident from the HFS. The reflection coefficient is approximately given by:

$$R_\mathrm{HFS}(\eta,\gamma) \approx \delta R_\mathrm{HFS}(\eta)(\gamma - 1)^2 \tag{11}$$

The absence of reflection of the waves incident from the HFS, predicted by the original Budden model, is obtained only for the degenerate case $\gamma = 1$.

The mode conversion coefficient for the LFS incidence depends both on the tunneling factor and the mismatch parameter. For a small mismatch of the FW wavelengths one obtains:

$$C_\mathrm{LFS}(\eta,\gamma) \approx C_\mathrm{B}(\eta) - \pi\eta e^{-2\pi\eta}(\gamma - 1) \tag{12}$$

where C_B is the Budden conversion coefficient. In contrast to Eqs. (10) and (11), the correction term in Eq. (12) is linear in the mismatch parameter. Note that for $\gamma < 1$, i.e. when the FW wavelength at the resonance side is greater than that at the cutoff side, the conversion coefficient exceeds the corresponding result of the Budden model. The maximum value of the conversion coefficient within the generalized Budden model

$C_{\max} = 48.6\%$ which is almost twice as high as the Budden limit of 25%, is achieved for $\gamma \approx 0.06$ and $\eta \approx 0.13$.

The triplet configuration model

According to the dispersion relation (2) for the FW, cutoffs of another type (the R-cutoffs) may appear at the plasma edge where the electron density is quite low. They are defined by the condition $R = n_{||}^2$. Usually there are two FW R-cutoffs inside the plasma. One is located at the LFS edge and affects the transfer of the electromagnetic energy from the ICRF antenna to the plasma. The problem of the coupling efficiency in ITER is expected to be solved by puffing some additional gas just in front of the antenna to increase the local plasma density.

The enhanced conversion efficiency in the presence of the R-cutoff located at the HFS was first identified numerically in [14]. Numerical simulations suggested a dramatic change in the behavior of the conversion coefficient: it was supposed to be an oscillatory function of all the parameters, which alters the location and thickness of the evanescence layer and/or the FW wavelength. An analytic model known as the triplet configuration was developed in [4], where the authors showed that the conversion efficiency in such a resonator-like structure was defined by the interference of the fast waves, one reflected from the conversion layer and the other from the R-cutoff. Thus the conversion coefficient depends on the phase difference between two reflected waves [4]:

$$(13) \qquad C = 2T_B(1 - T_B)\big(1 + \sin(2\Phi + \Psi)\big)$$

Within the triplet model one can, in principle, reach a total conversion by tuning the plasma parameters in such a way that the conversion layer is semi-transparent ($T_B = 1/2$) and provides opposite phases for the reflected waves.

The standing wave effect due to the constructive/destructive interference of the fast waves reflected from the conversion layer and the R-cutoff was experimentally observed on the JET tokamak [20]. Van Eester *et al.* showed that the RF power absorbed by electrons in (^{3}He)D plasma varied in an oscillatory way with the change of the minority ^{3}He concentration, in a fair agreement with the theory and the results of numerical modeling.

It should be noted that similar considerations were reported in papers preceding the formulation of the triplet configuration model. The effect of the closely spaced cutoff-resonance-cutoff triplet was discussed for the Alfvén resonance in [7], and the modification of the conversion efficiency due to the additional FW reflection from a perfectly-conducting metallic wall was studied in detail in [5].

Two mode conversion layers

Higher values of the conversion efficiency may be also obtained in a three-ion component plasma, in which two mode conversion layers are formed. The mechanism of the conversion enhancement is the same as for the triplet model, but the interference pattern in plasma is formed in a different way. The additional reflection of the FW occurs not from a poorly controlled R-cutoff but from the second conversion layer (Fig. 2). In this case the conversion coefficient is given by [10]:

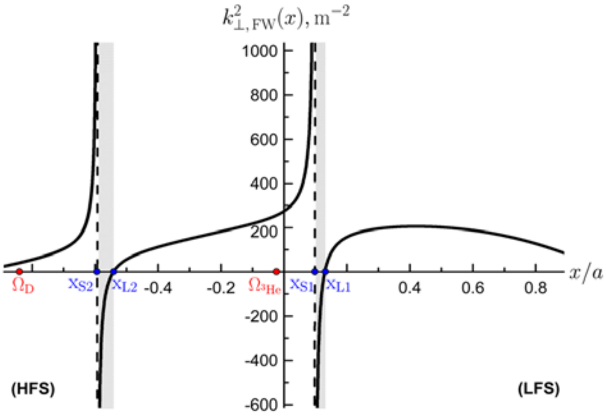

Fig. 2. A typical FW dispersion curve in a three-ion component plasma.

$$(14) \qquad C = T_1T_2(1 - T_1T_2) + 4T_1(1 - T_1)(1 - T_2) \cdot \sin^2(\Delta\phi/2)$$

The minimal value of the conversion coefficient corresponds to the Budden result, and the possible conversion enhancement is determined by the phase multiplier appearing in the second term in Eq. (14). For plasmas with two mode conversion layers the phase difference depends mostly on the distance between the conversion layers and can be changed by varying externally the plasma composition [10, 11].

An ICRF regime with two MC layers was observed on the JET tokamak during (^{3}He)H heating experiments [12, 16, 21]. The reason for the appearance of the second conversion layer in JET plasmas was an unavoidable plasma contamination by D-like intrinsic impurities such as carbon. The background level of C^{6+} ions before the shutdown of JET was sufficiently high so as to prevent the minority heating of deuterium in hydrogen plasmas [12, 16].

Recent (^{3}He)H experiments performed on JET were aimed at the optimization of the MC heating and the study of the ICRF-induced plasma rotation [21]. Depending on the minority ^{3}He concentration, two MC regimes were identified, one with a simultaneous presence of ^{3}He and D conversion layers and the other with a single D layer [21]. Different sensitivity of the heating efficiency to the change of ^{3}He concentration observed in these MC regimes can be qualitatively explained by the theory of Fuchs *et al.* [4] and its extension to the case of two mode conversion layers [10].

Conclusions

Although the mode conversion models discussed in this paper are simplified and do not take into account the actual 2-D geometry of tokamaks, they could serve as a good starting point for an effort to understand the basic underlying wave phenomena. Extension of the Budden model by inclusion of the additional FW reflection from the metallic wall, an R-cutoff or a supplementary conversion layer leads to the prediction of a possible enhancement of the conversion efficiency due to the constructive interference effect. These predictions were confirmed by a more sophisticated numerical modeling, and were also observed experimentally in (^{3}He)D and (^{3}He)H plasmas on JET. Such a heating enhancement is also expected to occur in the D-T plasma, which may be utilised to increase the fusion reactivity.

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