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Vlasov plasma description of pair plasmas with dust or ion impurities

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Abstract. The paper contains a unified treatment of disturbance propagation (transport) in the linearized Vlasov electron-positron and fullerene pair plasmas containing charged dust impurities, based on the space-time convolution integral equations. An initial-value problem for Vlasov-Poisson/Ampère equations is reduced to the one multiple integral equation and the solution is expressed in terms of forcing function and its space-time convolution with the resolvent kernel. The forcing function is responsible for the initial disturbance and the resolvent is responsible for the equilibrium velocity distributions of plasma species. By use of resolvent equations, time-reversibility, space-reflexivity and the other symmetries are revealed. The symmetries carry on physical properties of Vlasov pair plasmas, e.g., conservation laws. Properly choosing equilibrium distributions for dusty pair plasmas, we can reduce the resolvent equation to: (i) the undamped dispersive wave equations, (ii) wave-diffusive transport equation, and (iii) diffusive transport equations of oscillations. In the last case, we have to do with anomalous diffusion employing fractional derivatives in time and space.

Key words: pair plasmas • fractional diffusion • Vlasov plasmas

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Introduction

Electron-positron-dust/ion (e-p-d/i) plasmas can occur, e.g., in the inner region of accretion discs in the vicinity of black holes, in magnetospheres of neutron stars, in active galactic cores, and even in solar flare plasmas [2]. As for laboratory plasmas, it is known that p-e plasmas can be exited but do not allow life times sufficiently long for the excitation and developments of coherent structures like plasma waves and solitons. The annihilation time is short in comparison to the plasma period. This drawback is not present in the recently available long--lived pair plasmas composed of single charged fullerene pair plasma of molecules C^{+}_{60}, C^{-}_{60}, and electron-holes (e^{-}, h^{+}) in pure semiconductors also are pair plasmas if effective masses of electrons and holes are equal.

We reduce the initial-value problem of the standard Vlasov-Ampère/Poisson system of equations for multi-component plasmas, to the following multiple integral equation

$$E(x,t) = \int_0^t dt_1 \int_{-\infty}^{\infty} K(\xi,t_1)E(x-\xi,t-t_1)d\xi + G(x,t) \quad (1)$$

where

$$G(x,t) = -\sum_{\alpha}(q_{\alpha}/m_{\alpha})\int_0^t\int_{-\infty}^{\infty} u g_{\alpha}(u,x-ut_1)du dt_1,$$

$$K(x,t) = -\sum_{\alpha}\omega_{\alpha}^2 F_{0\alpha}(x/t) \quad \text{and} \quad \omega_{\alpha}^2 = \frac{N_0^{\alpha}q_{\alpha}^2}{\varepsilon_0 m_{\alpha}}$$

Detailed derivation of Eq. (1) can be found in [1, 4, 7].

Dispersion relations and exact solutions

The space-time convolution Eq. (1) can be solved by the use of a resolvent kernel $R(x,t)$. We shall write the solution as

$$(2)\quad E(x,t)=G(x,t)+\int_0^t dt_1\int_{-\infty}^{\infty} G(\xi,t_1)R(x-\xi,t-t_1)d\xi$$

where the kernel function $K(x,t)$ and the resolvent $R(x,t)$ satisfies the following resolvent equation

$$(3)\quad R(x,t)=K(x,t)+\int_0^t dt_1\int_{-\infty}^{\infty} K(\xi,t_1)R(x-\xi,t-t_1)d\xi$$

The last equation describes the plasma dynamic response $R(x,t)$ and its only dependence on the plasma equilibrium distribution. On the ground of Eq. (3), we note the time reversibility and space reflexivity. The important point to note is that according to the Noether theorem the properties are strictly related to energy and momentum conservation laws.

The time-Laplace and space-Fourier transforms of Eq. (3) lead to the usual dispersion relation of multicomponent plasmas – $D(k,s)$

$$(4)\quad R(k,s)=\frac{K(k,s)}{1-K(k,s)}\quad \text{and}\quad D(k,s)\equiv 1-K(k,s)=0$$

where $D(k,s)$ is the Fourier-Laplace symbol. In the case of diffusive transport equation of oscillations, the relation has no meaning. It is worth pointing out that the resolvent equation is more universal description of multicomponent plasmas than the usual dispersion relation.

Wave propagation

The advantage of the integral equations of Vlasov plasmas consists in obtaining the solutions separately composed of the forcing function $G(x,t)$ resulting from the initial value disturbance $g(u,x)$ and the resolvent kernel depending only on the plasma equilibrium $\sum_\alpha F_{0\alpha}(u)$. Assuming the hot components of pair plasma with the so-called "square" equilibrium velocity distributions and the cold heavy dust grains or ions, we have:

$$K(x,t)=-\omega_d^2 t\delta(x)-(\omega_g^2/2a)[H(x+at)-H(x-at)]$$

where $\omega_d^2 = N_d q^2/\varepsilon_0 m_d$ is for dust or ions, and the effective gap frequency is: $\omega_g^2 = (N_0 q^2/\varepsilon_0 m_0)(2-\nu)$. The constant ν is to ensure the charge neutrality of the plasma.

Hence the Fourier-Laplace symbols are

$$K(k;t)=-\omega_d^2 t-(\omega_g^2/ak)\sin(kat)\quad \text{and}$$

$$K(k;s)=-\frac{\omega_d^2}{s^2}-\frac{\omega_g^2}{s^2+k^2a^2}$$

The dispersion relation takes the form

$$D(k;s)\equiv s^4+s^2k^2a^2+s^2(\omega_d^2+\omega_g^2)+\omega_d^2k^2a^2=0$$

The respective dust-pair plasma wave equation for the resolvent kernel takes the form:

$$(5)\quad R_{tttt}-a^2R_{xxtt}+(\omega_d^2+\omega_g^2)R_{tt}-\omega_d^2a^2R_{xx}=0$$

The equation is a wave equation and the equation can be reduce to the simpler form of the dust/ion acoustic waves in the pair plasma with dust grains:

$$(6)\quad R_{tt}-c_s^2R_{xx}-(a^2/\omega_g^2)R_{xxtt}=0$$

Exact solution we can obtain substituting $s = -i\omega$ and since $\langle u^2\rangle = a^2/3$, we have the well-known Bohm-Gross dispersion relation, see also Ref. [2], $\omega^2 \approx \omega_0^2 + 3\langle u^2\rangle k^2$. We note that, that $K(x,t)$ and $R(x,t)$ are time reversible and x-space reflexive and the resolvent is an undamped dispersive wave, i.e. the Riemann function of the following dispersion equation

$$(7)\quad (a^2\partial_{xx}-\partial_{tt}+\omega_0^2)R(x,t)=0$$

The asymptotic expansion of the function is

$$R(x,t)\approx -\omega_0(4\pi Dt)^{-1/2}\sin(\omega_0 t+\pi/4)\quad t\to 0$$

where $D = 3\langle u^2\rangle/2\omega_0$. It appears that the asymptotic formula is common for all resolvents in case of equilibrium velocity distributions possessing all moments and the mean-square velocity being $\langle u^2\rangle$.

For Maxwellian plasmas, computer calculations, see Ref. [7], show that nature of plasma response is compound of a diffusive transition of oscillations and decreasing dispersive modes.

The next exact solution known to us is the resolvent for the Lorentz (Cauchy) electron-positron pair plasma. The equilibrium distribution is

$$F_{0e}(u)=F_{0p}(u)=\frac{\lambda}{\pi(\lambda^2+u^2)}$$

where λ is a positive number. The distribution is related to Lévy stable nongaussian processes and has no higher moments, e.g. mean-square velocity. It can be related to anomalous diffusion processes and is useful for modeling plasma with a high-energy tails that are typical in space plasmas.

Anomalous diffusion of oscillations

We quote new results concerning the resolvent for pair plasma with dust grains.

Let us describe the kernels due to equilibrium distributions of plasma species:

$$(8)\quad \begin{aligned} &K(k;t)=-\omega_d^2 t-\omega_g^2 t\exp(-|k|\lambda t),\\ &K(x,t)=-\omega_d^2 t\delta(x)-(\omega_g^2/\pi)\lambda/(\lambda^2+x^2/t^2)\end{aligned}$$

Introducing the parameter $\epsilon = \omega_d^2/\omega_g^2 < 1$, we can write

(9) $$R(x,t) = R_0(x,t) + R_d(x,t) + \sum_{n=1} \in^n R_n(x,t)$$

where $R_0(x,t) = -\frac{\lambda}{\pi} \frac{\omega_g t \sin \omega_g t}{(\lambda t)^2 + x^2} = -\omega_g \rho(x,t) \sin \omega_g t$.

The oscillating component with the dust plasma frequency is: $R_d(x,t) = -\omega_d \delta(x) \sin \omega_d t$.

The higher order terms due to the dust presence can be found and the analytical form can be presented. The resolvent $R(x,t)$ is drastically different from the previous one. It does not exhibit wave propagation and there is no dispersion relation. We observe a "diffusive transition" of oscillations.

If we assume the solution in the form

(10) $$R(x,t) = -\omega_0 \rho(x,t) \sin(\omega_0 t)$$

where $\int_{-\infty}^{\infty} \rho(x,t) dx = 1$.

We can show that the distribution function $\rho(x,t)$ solves a fractional partial differential equation, (fractional diffusion equation)

$$\partial_t \rho(x,t) = \partial_{|x|}^{\alpha} \rho(x,t)$$

The symmetric fractional derivative operator $\partial_{|x|}^{\alpha}$ corresponds to multiplication by the symbol $-|k|^{\alpha}$ in the Fourier space. For more details on symmetric α-stable (SαS) we refer the reader to [3, 6].

Conclusions

- An initial-value problem for Vlasov-Poisson/Ampère equations has been reduced to the integral equation and the solution to the problem is expressed in terms of a forcing function $G(x,t)$ and its convolution with a resolvent kernel $R(x,t)$.
- The forcing function is responsible for the initial disturbance and the resolvent is responsible for equilibrium distributions. Resolvent kernel equations are eligible for computer calculations.
- We have exhibited three types of exact closed-form solutions of the space-time resolvent equations. These solutions can be classified following the space-time behavior. The nature of plasma response is a compound of a diffusive transition, see Eq. (10), being essentially a plasma oscillation mode with the ω_0 – plasma frequency and the diffusive types of amplitude envelop, and a decreasing dispersive wave mode.
- The crucial point of the paper is the relation between equilibrium distributions of plasma species and the type of propagation or diffusive transition of plasma response to a disturbance.
- Dust/ion impurities may cause appearance of dust or ion acoustic waves and solitons. They disturb oscillations but the diffusive transitions remain unchanged according to envelop $\rho(x,t)$.
- There is a suggestion that the envelopes of diffusive transition of oscillations can be governed by a symmetric α-stable (SαS) process. The probability distributions of the processes are related to the fractional diffusive transition described by the fractional diffusion equations.
- Up to now, the necessary and sufficient conditions of the type of disturbances propagations have not been determined.

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