NUKLEONIKA 2013;58(2):281–285

ORIGINAL PAPER

A new method of determining the parameters of thermonuclear plasma on the basis of multichannel polarimetric measurements

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Abstract. A gradient method has been proposed as an effective procedure to reconstruct real parameters of tokamak plasma by means of multichannel polarimetric measurements. High efficiency of the suggested procedure is illustrated by numerical calculations performed for a given plasma model taking into account the poloidal component of magnetic field. Polarization state of electromagnetic wave traditionally is characterized by azimuthal – ψ and ellipticity – χ angles. Evolution of these parameters along the ray is described by the equations of angular variables technique (AVT) introduced in previous works of the present authors. Numerical simulations have approved that the gradient procedure provides acceptable accuracy of inversion already after several iterations.

Key words: angular variables • gradient method • plasma diagnostics • plasma polarimetry • polarimetric data inversion

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Received: 16 July 2012
Accepted: 8 January 2013

Introduction

There is a growing interest in developing a reliable method for the measurement of the internal magnetic field at high temperature, magnetically confined plasmas. A special need for such diagnostic arises in the investigation of the tokamak devices in which the measurement of the poloidal field distribution would yield the plasma current density profile. This information is essential for understanding confinement, stability and energy balance of the tokamak plasma.

A precise control of the plasma position is a key issue in order to avoid damages on the first wall of the device. Such a control is essential when high-power long-duration plasmas have to be performed as on the Tore Supra tokamak. The current carried by the plasma can be localized using magnetic measurements (pick-up coils) outside the plasma. The plasma boundary can thus be identified and controlled in a real time in less than a few milliseconds. In a tokamak plasma the distribution of the plasma current plays an important role because the resulting poloidal magnetic field determines the confinement properties and is crucial for the stability of the tokamak plasma. The appearance and growth of numerous instabilities are closely connected to the existence of certain rational surfaces in the plasma as well as subtle local modifications of the poloidal magnetic field.

In order to get information on the current distribution inside the plasma, more sophisticated calculations must be performed. Because magnetic measurements are no longer sufficient to constrain the solution when detailed information on current distribution inside the plasma are mandatory, other measurements must be introduced as external constraints in the solver. At the beginning of 2008, a five-channel vertical DCN – 1.58 THz (deuterium cyanide) laser system has been constructed on the EAST device using both the interference and polarization effects. The system is used mainly for getting electron density profile, poloidal current profile and for density feedback control simultaneously.

The use of polarization phenomena in the plasma diagnostic, encountered in practice, is the major difficulty. It results from the fact that in the general case when the effects of Faraday and Cotton-Mouton determining the state of polarization of the sample beam are comparable, the equations showing the changes of the measured polarization angles, depending on plasma parameters are partial differential equations [3, 9]. As a result, the lack of simple analytical relationship between changes in the polarization state of the beam sample, and the plasma parameters, forces us to use unconventional methods to reproduce plasma parameters on the basis of experimental results.

Majority of plasma models, used for polarimetric data inversion deal with a very simple, if not primitive, configuration of electron density and static magnetic field [4]. The papers [2, 7, 8] have studied a "non-conventional" procedure for polarimetric data inversion. This procedure has implied fitting the two-parameters plasma model with toroidal magnetic field to experimental values of angular parameters of polarization ellipse. In the present work, as in previous articles [2, 7, 8], we use a new approach in plasma polarimetry: angular variables technique (AVT), which deals with angular parameters of polarization ellipse. This allows simplifying the problem and avoiding Stokes vector formalism.

This paper extends the gradient approach, used in [2, 7, 8] only for two polarimetric data, for a wider set of polarimetric parameters (2 polarimetric channels that is 4 polarimetric parameters) and simultaneously involves magnetic data into the gradient procedure.

Gradient algorithm for experimental data inversion in frame of fourth parameters plasma model

Polarization state of electromagnetic wave traditionally is characterized by azimuthal – ψ and ellipticity – χ angles. Evolution of these parameters along the ray is described by the equations of angular variables technique (AVT) [3]:

$$\frac{\partial\psi}{\partial\sigma} = 0.5(\Omega_2 - (\cos[2\psi[\sigma]\Omega_1 + \sin[2\psi[\sigma]]\Omega_2)\tan[2\chi[\sigma]])$$

$$(1) \quad \frac{\partial\chi}{\partial\sigma} = 0.5(\sin[2\psi[\sigma]\Omega_1 - \cos[2\psi[\sigma]$$

$$\frac{\partial\chi}{\partial\sigma} = 0.5(\sin[2\psi[\sigma]]\Omega_1 - \cos[2\psi[\sigma]]\Omega_2)$$

Here, σ are the lengths along the ray and $\Omega_{1,2,3}$ are the plasma parameters, widely used in plasma polarimetry [8], coefficients $\Omega_{1,2}$ corresponds to Cotton-Mouton effects, and coefficient Ω_3 – to the Faraday phenomenon [7, 8, 10]:

$$(2) \quad \begin{aligned} \Omega_1 &= C_1\lambda^2(B_x^2 - B_y^2)N_e \\ \Omega_2 &= C_2\lambda^2 2(B_x B_y)N_e \\ \Omega_3 &= C_3\lambda^2 B_z N_e \end{aligned}$$

within the SI system the constants C_i are: $C_1 = 2.45681 \times 10^{-11}$; $C_2 = 2.45681 \times 10^{-11}$; $C_3 = 5.26241 \times 10^{-12?}$, respectively.

It is noteworthy that AVT Eqs. (1) are equivalent to the equations of Stokes vector formalism (SVF) [9], but are much more convenient as compared to SVF equations: two equations in AVT instead of three equations in SVF.

Let $\psi(p)$ and $\chi(p)$ are solutions of AVT Eqs. (1) for i-th polarimetric channel and $p = (p_1, p_2, \ldots, p_N)$ is a set of N plasma parameters to be determined by fitting angular variables ψ and χ to experimental polarimetric data.

Equating $\psi_i(p)$ and $\chi_i(p)$ to experimental observations $\psi_{i,\exp}$ and $\chi_{i,\exp}$ we obtain $2i$ equations for N parameters $(p_1, p_2, \ldots, p(N))$

$$(3) \quad \psi_i(p) = \psi_{i,\exp};\ \chi_i(p) = \chi_{i,\exp}$$

In general, we do not have analytical solutions of AVT equations, so we are enforced to apply one of the numerical methods, namely – gradient approach [5, 6]. Let us introduce the error function $\phi(p)$ which is quadratic measure of inconsistency between theoretical and experimental values

$$(4) \quad \phi^{(k)} = \sum_{i=1}^{n}(\psi_i(k) - \psi_{i,\exp})^2 + (\chi_i(k) - \chi_{i,\exp})^2$$

where: $\phi^{(k)}$ – error function for the k-th step; $\chi_i^{(k)}$ – polarization angles for the i-th channel at the k-th step; $\psi_{i,\exp}$ – experimental values of the polarization angles obtained for the i-th channel.

In consequence, the equation

$$(5) \quad \phi(p) = 0$$

which minimizes inconsistency between theoretical and observation data, is equivalent to Eq. (3).

Let $p_i(0)$ be starting value of parameter p_i in the procedure of consequent approximations. According to the gradient method, increment $\delta_i^{(1)}$ at the first step should be proportional to gradient $\nabla\phi$ of the error function taken with a minus sign:

$$(6) \quad \delta_i(1) = -H_i(1)\, G_i(1)$$

Here G means the gradient of the error function

$$(7) \quad G = \nabla\phi(p_1, p_2, \ldots, p_n)$$

whose components are respectively:

$$(8) \quad \frac{\partial\phi}{\partial p_i} \approx \frac{\Delta\phi}{\Delta p_i} = \frac{\phi(p_1,\ldots,p_2 + p_i,\ldots,p_n) - \phi(p_1,\ldots,p_i,\ldots,p_n)}{\Delta p_i}$$

It is reasonable to choose the coefficient H_i according to inequality

$$(9) \quad H_i \le \frac{1}{2M}\frac{\phi(p_0)}{G_i^2}$$

where M is total number of parameters, envisaged for fitting.

As a result

$$\delta_i(1) \le -\frac{1}{2M}\frac{\phi(p_0)}{G_i} \quad (10)$$

so that increment

$$\Delta\phi(1) \le \sum_{i=1}^{M}\frac{\partial\phi}{\partial p_i}\delta_i(1) \quad (11)$$

of the error function $\phi(p)$ will not exceed

$$\Delta\phi(1) \le \sum_{i=1}^{M}\frac{\partial\phi}{\partial p_i}\delta_i(1) = \frac{1}{2}\phi(p_0) \quad (12)$$

Such increment provides monotonous decreasing of the error function its exponential convergences to zero.

Thus, the first iteration for parameter p_i will be

$$p_i(1) = p_i(0) + \delta_i(1) \quad (13)$$

Repeating these operations with the starting value $p_i(1)$, we arrive to the second iteration $p_i(2)$, then to $p_i(3)$ and so on. According to general theory, sequence $p_i(1)$, $p_i(2)$, …, $p_i(N)$ tends to solution of Eqs. (3) and (5).

In the papers [2, 7, 8] the gradient procedure was applied for polarimetric data inversion in conditions of only two parameters: $M = 2$. In the next section we shall perform numerical simulations for more complicate model with $M = 4$ parameters.

Numerical simulations

In order to demonstrate the correctness of the gradient method the simple model of plasma configuration has been chosen: plasma with a circular cross-section of the magnetic flux surfaces and parabolic density profile $N_e = N_0(1 - \rho^2)$, where $\rho = r/a$ is the normalized radius of the flux surface in the plasma with a minor radius a (Fig. 1). In the case of a large aspect-ratio circular plasma $a << R$, with a toroidal current density distribution $j = j_0(1 - \rho^2)^\nu$ providing the total current j_0, the magnetic field components, at the point with the radius R and normalized radius ρ are [10]:

$$B_R = B_0\frac{R_0}{R}$$
$$B_\theta = \frac{\mu_0 I_0}{2\pi a}\frac{1-(1-\rho^2)^{\nu+1}}{\rho} \quad (14)$$

where B_R and B_θ are the toroidal and the poloidal magnetic fields and B_0 is the toroidal magnetic field on the magnetic axis of the plasma with the major radius R_0.

The plasma parameters have been chosen similar to that of the large thermonuclear plasma devices, like JET or ITER: $B_0 = 6$ T, $R_0 = 3$ m, $a = 1.80$ m, $N_e = 0.75 \times 10^{20}$ m^{-3}, $I = 3.2$ MA, $\nu = 2.6$. For such cords, the magnetic field components in the beam reference frame are: $B_x = B_R$, $B_y = B_\theta \sin\varphi$ and $B_z = -B_\theta \cos\varphi$.

Accepting configuration of two vertical polarimetric channels (Fig. 1), we place the sources (DCN lasers with a wavelength of $\lambda = 195$ μm and initial polarization $\psi_0 = \pi/4 \cdot \chi_0$) and receivers at the points

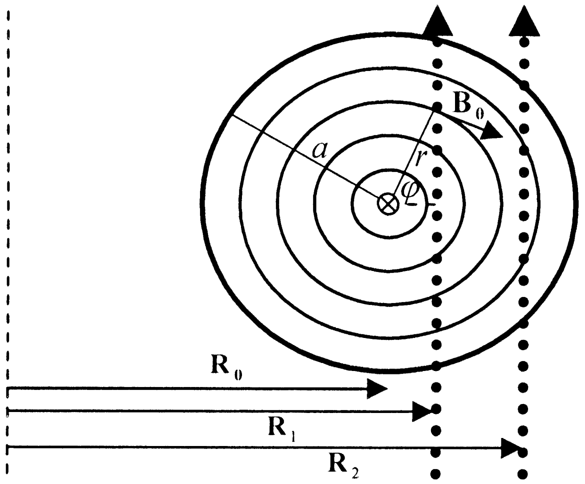

Fig. 1. Schematic of a vertical lines of sight in the plasma with circular flux surfaces.

$$R_1 = 0.3 \text{ m}; \;\; R_2 = 0.8 \text{ m} \quad (15)$$

in a poloidal plane of a tokamak.

According to Eqs. (1), "experimental" values obtained for the polarization angle for the corresponding canals are:

$$\psi_{1,\exp} = 1.16585; \;\; \psi_{2,\exp} = 1.451516;$$
$$\chi_{1,\exp} = 440\,777; \;\; \chi_{2,\exp} = 0.272166 \quad (16)$$

Now, our task is to reproduce the real (of interest to us) plasma parameters on the basis of these data obtained from polarimetric measurements, using the gradients method.

Based on the genuine parameters p_i we calculate polarimetric variables $\psi_i(p)$ and $\chi_i(p)$ from AVT Eq. (1) and consider them as "experimental" data $\psi_{i,\exp}(p)$ and $\chi_{i,\exp}(p)$. Total current I in Eq. (14) we also identify with the experimental value, obtained from magnetic measurements. Toroidal magnetic field, having only z-component, we also assume to be known from magnetic measurements.

Then, we take starting parameters $p_i(0)$ and perform gradient procedure of consequent approximations, described in section 'Gradient algorithm for experimental data inversion in frame of fourth parameters plasma model' "genuine" p_i and starting values are presented in Table 1 along with the consequent approximations $p_i(1), p_i(2), …, p_i(N)$. Table 1 presents also evolution of the error function ϕ.

Starting parameters $p_i(0)$ were chosen to be about 30% lesser than the "genuine" ones to test the convergence of gradient method to proper values. The first and second columns in Table 1 show parameters p_i and their units. "Genuine" parameters p_i are present in the third column.

It follows from Table 1 that values $p_i(7)$ achieved at the ninth step of iteration procedure differ from "true" values p_i^* at most 2–3%.

Changes of the azimuthal ψ and ellipticity angle χ along the radius during several selected steps obtained in the first and second channel are presented in Figs. 2–5.

Table 1. Evolution of plasma parameters p_m in the frame of gradient procedure, applied for polarimetry data inversion

Parameters p_i	Unit	p_i^*	Consequent approximations				
			$p_i(0)$	$p_i(1)$	$p_i(2)$	$p_i(6)$	$p_i(7)$
$p_1 = N_e$	10^{20} m^{-3}	0.75	0.5	0.742	0.763	0.753	0.75
$p_2 = I$	MA	3.2	3.0	3.028	3.34	3.22	3.21
$p_3 = a$	m	1.8	1.5	1.54	2.02	1.78	1.8
$p_4 = \nu$	1	2.6	2.0	2.1	2.33	2.57	2.58
Error function ϕ	1	0.00	0.2428	0.0272	0.009	5×10^{-5}	5.86×10^{-7}

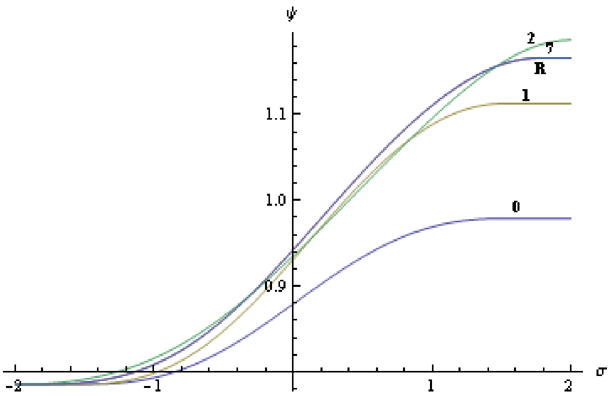

Fig. 2. Changes of the azimuthal angle ψ along the radius during several successive approximations (0 – initial conditions, *R* – the real plasma parameters corresponding to the measurement results) obtained in the first channel.

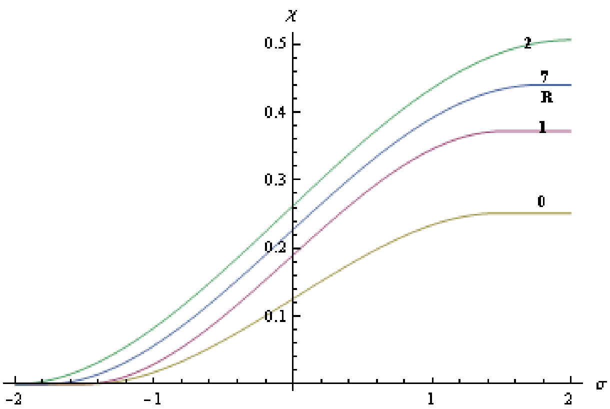

Fig. 3. Changes of the ellipticity angle χ along the radius during several successive approximations (0 – initial conditions, *R* – the real plasma parameters corresponding to the measurement results) obtained in the first channel.

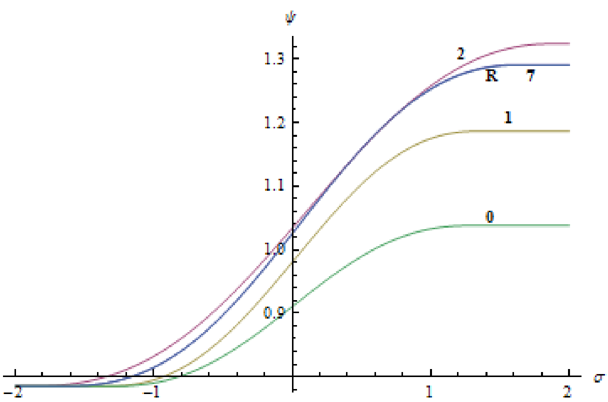

Fig. 4. Changes of the azimuthal angle ψ along the radius during several successive approximations (0 – initial conditions, *R* – the real plasma parameters corresponding to the measurement results) obtained in the second channel.

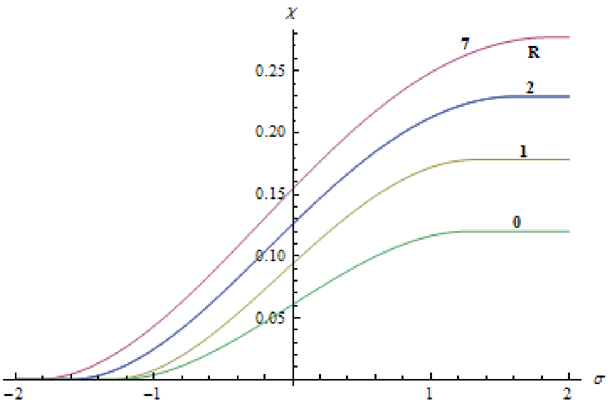

Fig. 5. Changes of the ellipticity angle χ along the radius during several successive approximations (0 – initial conditions, *R* – the real plasma parameters corresponding to the measurement results) obtained in the second channel.

Analysis of these graphs (Figs. 2–5) shows the full correlation between the error function ϕ, and the variability of the two polarization angles ψ, χ, which are becoming more and more similar to the conditions set out in the experiment.

Conclusion

The need for maximum reliable data on the parameters of nuclear plasma forces to search for new methods applied in diagnostics. At first, the equilibrium reconstruction code [1] used in tokamak experiments (EFIT at JET) to map diagnostic information and to derive basic plasma properties like current density and safety factor was based on magnetic probe measurements only. On the basis of the resulting data, the system was able to solve the Grad-Shafranov equation by adjusting the flux fit function. Naturally, the accuracy of the results depended on the accessibility and quality of the diagnostic information. However, as shown by the experimental data obtained exclusively by means of magnetic measurements do not allow for accurate reproduction profiles of the safety factor and current density in particular plasma scenarios. Therefore, it became necessary to supplement the data thus obtained by the additional internal diagnostic information. One of the elements of such additional system is setup for the interferometer-polarimeter which allows a measurement of the line integrated density and the Faraday rotation along the same straight line. Incorporating the Faraday rotation mainly changes the plasma elongation. In shear reversed plasma incorporating the Faraday rotation data as an additional constraint results in an increase of the safety factor by 10%. However, the use of pure Faraday rotation causes a distinct restriction of the measurement, and, therefore, in practice (for example JET) may be used a few channels only.

In the proposed method it has been taken into account both the influence of the Faraday rotation and the Cotton-Mouton effect. This allows to use practically all channels for polarimetric measurements. It is important, therefore, that each additional channel allows to determine two additional parameters of the plasma. The problem with the lack of analytical relations between the measured polarization angles and the changes of plasma parameters that occurs with the general case, is solved by applying the gradient method, which allows for rapid reconstruction of plasma parameters with practically any accuracy.

This paper considers a four-parameter plasma model only as illustration of a multiparameter approach. In fact, the amount of parameters can be increased or reduced in dependence on the aim of plasma modelling. If we would like to describe fine details of plasma configuration we may involve slight asymmetry in the density distributions both in the horizontal and vertical directions, to account a potential influence of toroidal form of camera or an influence of diverter area.

Additional parameters in electron density distributions can be used to fit the model to the Thomson scattering data, which bear information on local density variations. Of special interest are additional parameters, which might describe plasma configuration in the diverter area.

Summarizing the results of numerical simulations, we may say that the gradient procedure has approved its efficiency in condition of a four-parameter model it has demonstrated sufficiently fast convergence to "genuine" data and acceptable accuracy. Of course, accuracy of the gradient procedure in conditions of real plasma, which is not necessarily close to a chosen model requires further studies. As was mentioned above, the starting parameters $p_i(0)$ were chosen to be 30–50% larges or lesser than the "genuine" ones only to verify their convergence to the "genuine" parameters and to test the gradient method. In practice, having available the results obtained by other methods, we can always choose the starting point much closer to the actual conditions, which of course greatly reduce the number of necessary steps, so in consequence, we will focus exclusively on the measurement accuracy.

Therefore, we have reason to believe that the presented method should lead to a greater use of polarimetry in the diagnostic of thermonuclear plasma making it an effective instrument for the comprehensive diagnostic applied in tokamaks.

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