NUKLEONIKA 2013;58(4):533–536

ORIGINAL PAPER

An improved formula for dead time correction of G-M detectors

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Abstract. Different analytical formulae have been described in the literature to modify response of Geiger-Müller (G-M) detectors. In this work, improvement of a previously proposed dead time correction formula was investigated. A set of experimental data of a decaying source was the basis of the analysis. A general agreement is seen with the experimental data. The result was compared with those obtained by the original work. Numerical aspects were also examined.

Key words: dead time model • Geiger-Müller (G-M) detector • decaying source experiment • hybrid model

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Received: 25 June 2013
Accepted: 20 September 2013

Introduction

The Geiger-Müller (G-M) counters exist in nuclear physics for already almost 100 years. They have a range of applications (e.g. survey dosimeters). Because they need a simple and low cost electronic circuitry, these detectors are perfect devices when there is no need for incident radiation energy measurement. Also, the problem of finding dead time corrections, at least in the first two approximations, mentioned in the paper, appeared very soon and became typical student's problem already for a few decades.

Nonlinear response to the radiation intensity at high counting rates is the only drawback of these detectors. This is due to the dead time effects. Hence, different investigations have been conducted to tolerate this problem. Two extreme dead time models of G-M detectors are paralizable and non-paralizable models [6]. Efforts have been made to extend the range of usefulness of the detector [1, 4, 5, 7, 9–11]. A hybrid model, which shows a good consistency with experimental data, is proposed by Lee and Gardner [8]. Because of complexity of the dead time process in G-M detectors, this model also shows significant deviations at high counting rates. This paper is focused on error reduction of hybrid model formula based on its nonconformities to the experimental data.

Of course, the presented formulae extend the applicability of G-M counters to higher intensities. Nevertheless, the question is, why it should be important? Usually, any correct monitoring system should work in linear regime meaning that the achievement of this goal is very simple (e.g., increase of the distance between the radioactive source and G-M counter, or diminish activity of the source or, place the counter in a shield equipped with a small collimator). In addition, in the situation where we are dealing with an extremely large diapason of intensities, we can use rather few detectors of different efficiencies. When standard monitors working normally in linear regime are unexpectedly highly irradiated, the readout of G-M counters might show a great systematic error. For example, during nuclear reactor accidents real radiation intensity must be evaluated from detectors working far out of their normal regime. Therefore, very detailed mathematical analysis of rather abnormal working regime of the detector would be required for the readers.

Hybrid model

Figure 1 illustrates three different models. Figure 1a indicates the non-paralizable dead time behaviour. A fixed duration, τ_n, is assumed to follow each true event which occurs during live period of the detector. Through τ_n, no interaction can start an avalanche in the detector. Therefore, three interactions are occurred, but two pulses are generated. One of them is lost due to the non-paralizable dead time effect. Figure 1b shows paralizable dead time behaviour. Paralizable dead time, τ_p, is refreshed by any incident radiation, which interacts during τ_p. Note that the notation "Tc" means true count, which can be recorded by the detection system, and the notation "Lost" refers to interactions, which are lost because of dead time of the detector. Figure 1c shows hybrid model suppositions. Each detection process starts with a non-paralizable dead time and is followed by the paralizable one. The detector will not record any interaction during non-paralizable dead time. The total process is refreshed by any interaction during paralizable dead time. Equation (1) indicates hybrid model formula [8]. Its constant parameters are determined by curve fitting of this equation to a set of experimental data.

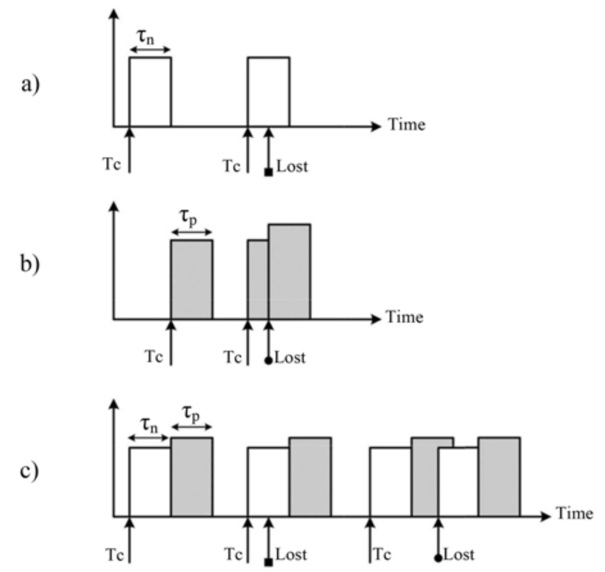

Fig. 1. Illustration of different dead time models behaviour. (a) non-paralizable model, (b) paralizable model, (c) hybrid model.

$$m_{\text{Hybrid}} = \frac{n e^{-n\tau_p}}{1 + n\tau_n} \tag{1}$$

where: m – observed count rate, n – true count rate, τ_p – paralizable dead time, τ_n – non-paralizable dead time.

Experimental data

To check the validity of the results, experimental verification is essential. Experimental data of a decaying source are used for this purpose. An activated Mn^{56} gamma source is placed in the front of a G-M tube and measurements are performed every ten-second [8]. The experimental data are shown in Fig. 2. Vertical axis of the figure is in a logarithmic scale. Noticeably, the half--life of Mn^{56} is 2.578 h ($\lambda_{Mn^{56}} = 7.4686 \times 10^{-5}\ s^{-1}$). For decay processes, the relation between the true count rate, $N(t)$, decay constant of the radioactive source, λ, and the background level, N_{BG}, is given in Eq. (2)

$$N(t) = N_o e^{-\lambda t} + N_{BG} \tag{2}$$

N_o and N_{BG} are the constants which are determined by fitting of this equation to the experimental data. Dashed line in Fig. 2 shows $N(t)$, the fitted curve to the experimental data. It is clearly seen that the observed count rates are deviated from the true counting curve, $N(t)$, due to the dead time effects during the first 5×10^4 s of the measurements. This experimental data and the fitted curve are the bases of the validations given in the following sections. Further information about decaying source experiment is given elsewhere [7, 8].

Methods and results

Improved dead time formula (IDTF)

Hybrid model formula (Eq. (1)) is a two-degree of freedom equation. In fact, dead time phenomenon is a

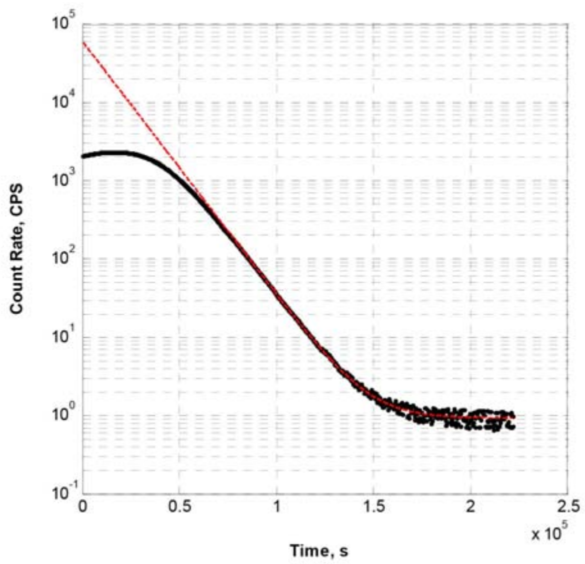

Fig. 2. Bullet points are experimental data of a decaying source. The dashed line is fitted curve of Eq. (2) to the experimental data ($N_o = 6.11 \times 10^4$, $N_{BG} = 0.932$, $\lambda = 7.4686 \times 10^{-5}\ s^{-1}$).

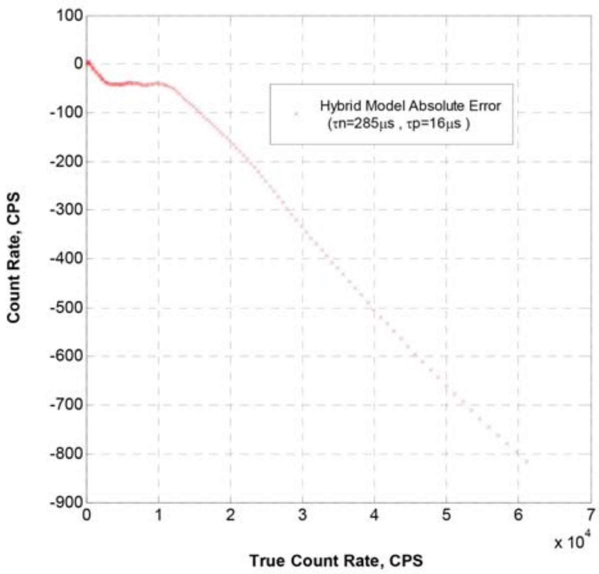

Fig. 3. Absolute error of hybrid model.

complicated process. Therefore, details of the process cannot be described by simple assumptions of hybrid model shown in Fig. 1c. Figure 3 shows the plot of the absolute error of hybrid model. The curve is mostly similar to a linear line.

In order to have a better description of the dead time process in G-M tubes, an improvement is required for hybrid model regarding its deviation from experimental data. Equation (3) shows the basic assumed relation between the hybrid model formula and its improved form.

(3) $$m_{\mathrm{IDTF}} = m_{\mathrm{Hybrid}} + F_{\mathrm{D}}$$

The last term, F_{D}, is the remainder or absolute error of hybrid model (Fig. 3). The shape function of the remainder is linear. Therefore:

(4) $$F_{\mathrm{D}} = \theta n$$

In this equation, θ is a constant. Now, formulation for the dead time can be rewritten as:

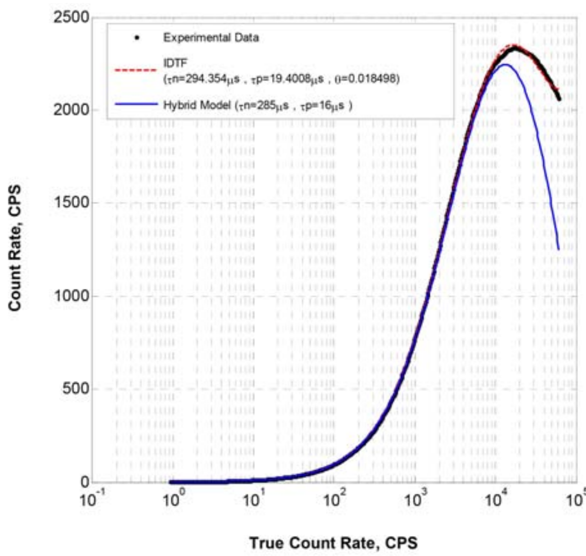

Fig. 4. Fitting curves of hybrid model and IDTF to the experimental data.

(5) $$m_{\mathrm{IDTF}} = \frac{n e^{-n\tau_p}}{1+n\tau_n} + \theta n$$

Parameters of this equation can be estimated by using a curve-fitting method. Related material or helps on numerical solution of the problem might be available upon request for the readers. Figure 4 shows the fitting curves of hybrid formula and IDTF formula (Eqs. (1) and (5)) with the experimental data. Fitted parameters are shown by the figure. Obviously, the improved form of hybrid model shows a general agreement with the experimental data. A better comparison of the models is shown in Fig. 5. Vertical axis of this figure is the point-to-point division of the model calculated count rate by the experimental data. The ideal curve must be uniform over the entire rate range of count rates. Below 10 cps, due to the large statistical fluctuations, great deviations are illustrated. This is a systematic error. Both models show nearly the same response up to 10^4 cps. At high-count rates, hybrid formula is underestimated. It declines to less than –40% at 6×10^4 cps. The IDTF has a good agreement with the experimental data in the whole range.

Numerical analysis

In previous section, the hybrid model and its modified form were compared based on the data from a decaying source experiment. Constant parameters of the formulae were determined by curve fitting. This section assesses the numerical aspects of the problem for correction purposes.

As it was described previously, the observed counting rates of G-M tubes show a nonlinear response to the incident radiation intensity, which must be corrected by using an appropriate dead time correction method. Therefore, in practice, an iterative method of numerical solution must be applied to the implicit formulae. Hybrid formula and IDTF are also implicit formulae. A popular method for solving nonlinear implicit equations is the fixed-point iteration method [2, 3]. Rearranging Eq. (1), the recursive formula for hybrid model can be written as

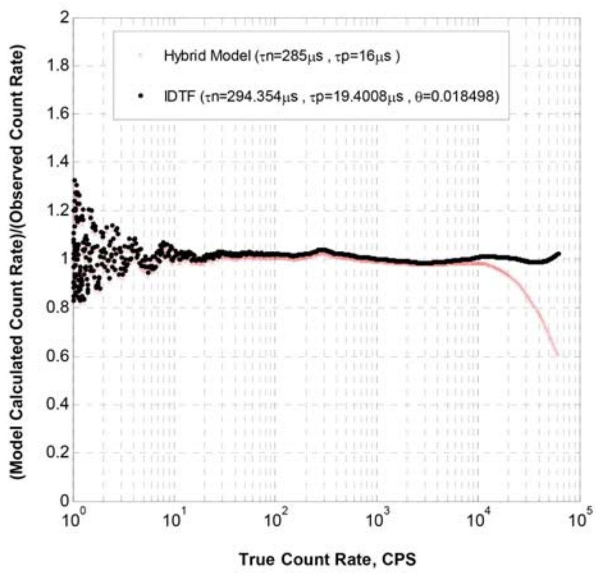

Fig. 5. Point-to-point division of model calculated count rate by the observed count rate.

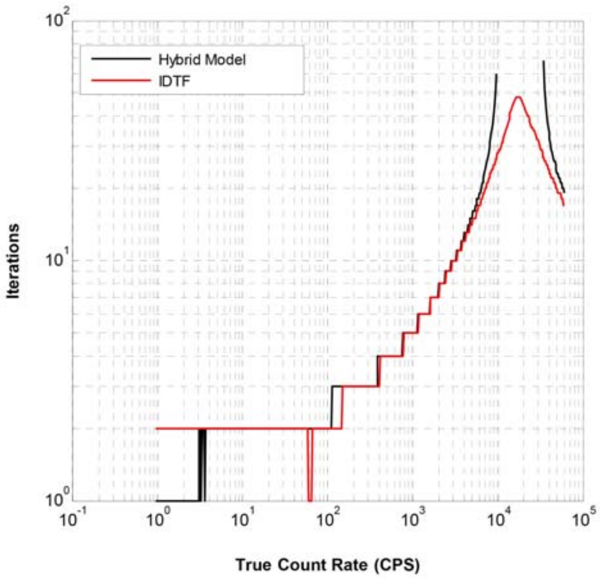

Fig. 6. Number of numerical iterations needed for correction.

$$n_i = m(1 + n_{i-1}\tau_n)e^{n_{i-1}\tau_p} \tag{6}$$

Similarly, the improved hybrid model formula can be rearranged as

$$n_i = \frac{m}{\dfrac{e^{n_{i-1}\tau_p}}{(1 + n_{i-1}\tau_n)} + \theta} \tag{7}$$

In these equations, i is the index of the iteration number. For $i = 1$, n_0 should be initialized. The value of m is set as the initial point.

The convergence of the models is investigated by using these recursive formulae. The results are exhibited in Fig. 6. Because the slope of the experimental data at high-count rates (around 1.5×10^4 cps) is very small (Fig. 4), the numerical correction shows instability at these regions. Considering the number of needed iterations to reach the convergence criterion, the hybrid formula and IDTF show the same behaviour except for high-count rates. A slightly better performance at high-count rates is seen for IDTF. An online application of dead time correction of G-M counters needs calculation of the answer immediately after each measurement.

Conclusion

Based on the absolute error shape function, the hybrid formula was improved. In addition, validation of the improvement was checked by using a set of experimental data of a decaying source. The results were given in Figs. 3 to 5 elaborately. The improved hybrid model showed a good agreement with experimental data, while the deviation for hybrid model decreases to less than –40% at 6×10^4 cps. Numerical assessment of the models was performed by using fixed-point iteration method. The results were shown in Fig. 6. The number of needed iterations to reach the convergence criterion is important because it directly relates to the amount of time to find the solution by the numerical method. A little better behaviour is seen by IDTF at high-count rates.

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