



Reconstruction of hit time and hit position of annihilation quanta in the J-PET detector using the Mahalanobis distance

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Abstract. The J-PET detector being developed at the Jagiellonian University is a positron emission tomograph composed of the long strips of polymer scintillators. At the same time, it is a detector system that will be used for studies of the decays of positronium atoms. The shape of photomultiplier signals depends on the hit time and hit position of the gamma quantum. In order to take advantage of this fact, a dedicated sampling front-end electronics that enables to sample signals in voltage domain with the time precision of about 20 ps and novel reconstruction method based on the comparison of examined signal with the model signals stored in the library has been developed. As a measure of the similarity, we use the Mahalanobis distance. The achievable position and time resolution depend on the number and values of the threshold levels at which the signal is sampled. A reconstruction method as well as preliminary results are presented and discussed.

Key words: J-PET detector • Mahalanobis distance • time resolution • threshold levels

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Received: 5 July 2015, Accepted: 28 August 2015

Introduction

A new concept of large acceptance the Jagiellonian positron emission tomograph (J-PET) based on the polymer scintillators [1–9] has been developed recently, which provides a solution for a whole-body positron emission tomograph (PET) imaging with an improved time of flight (TOF) resolution¹, as it relies more on the precise time measurement than on energy resolution (like in traditional PETs) and the signals that we probe are very narrow few nanoseconds. To this end, a dedicated new electronics was developed, which enables signal sampling in voltage domain with a time precision of about 20 ps [13]. Moreover, we have been developing new methods of gamma quanta hit time and hit position reconstruction to fully exploit potential of the new tomograph [11, 14, 15]. One of the developed methods is based on the comparison of measured signals with respect to a library of synchronized model signals registered for a set of well-defined positions of scintillation points. The hit position is reconstructed as the one corresponding to the signal from the library that is most similar to the measured signal [11]. The measured signal can also be compared to the averaged model signals determined for each scintillation point that speeds up significantly the reconstruction [12]. The degree of similarity between the measured and the reference signals from the library can be

¹ In current tomographs, the best TOF resolution was achieved with LSO (Lu₂SiO₂) crystals and it is equal to about 400 ps [10] and in our previous studies with the plastic scintillators, ~290 ps was achieved [11, 12].

expressed by several metrics, e.g. the methods described in the articles [11, 12] use χ^2 metric. In the present article, we discuss a method based on the Mahalanobis distance [16], which accounts for the correlation among the compared variables.

Principle of time and position reconstruction using library of model signals

The method was validated by using the experimental data collected by means of the double-strip J-PET prototype and ^{22}Na isotope as a source of annihilation gamma quanta. For noise suppression and selection of annihilation gamma quanta, a coincident registration of signals from both detectors was used. Pedestal correction was also implemented to all the collected signals. In order to construct the library only with the high-energy deposition events, only those signals were selected for which number of registered photoelectrons is greater than half of the number of photoelectrons corresponding to the Compton edge for 511-keV gamma quanta. Next, a two-dimensional plot of number of photoelectrons obtained from left- and right-sided photomultipliers connected to the same strip was bisected into four regions (region 1, region 2, region 3, and region 4). An example for central position is shown in Fig. 1. Bisection was done in order to suppress the uncertainties arising due to the variation of time resolution as a function of number of registered photoelectrons. Then, for each bisected region, signals were synchronized by shifting their time scales in such a way that time of the gamma quantum hit inside the detector is the same for all events in the library. Finally, the shape of a model signal for each bisected region is determined by averaging pedestal corrected and synchronized signals².

For reconstructing the hit-position of gamma quanta in the detector, the measured signals are compared with each model signal from the library. Signals can be represented as a vector of time values at defined threshold levels. In this article, we consider two threshold cases (-60 mV and -120 mV) where t_{L1} , t_{L2} and t_{R1} , t_{R2} are time values on left and right signals (registered simultaneously at both ends of the scintillator) defined at -60 mV and -120 mV threshold levels, respectively:

$$(1) \quad \vec{t} = [t_{L1}, t_{L2}, t_{R1}, t_{R2}].$$

The position and time of the examined signal is determined by minimizing the Mahalanobis distance (Eq. (2)) between this signal and the model signals stored in the library. For each

$$(2) \quad M(z, \delta t, \text{region}) = \sqrt{\begin{bmatrix} \bar{X}(\delta t, z, \text{region}) \\ \cdot [Cov(z, \text{region})]^{-1} \\ \cdot \bar{X}(\delta t, z, \text{region}) \end{bmatrix}}$$

² For detailed descriptions of experimental setup and the method of constructing library of model signals, see Ref. [12].

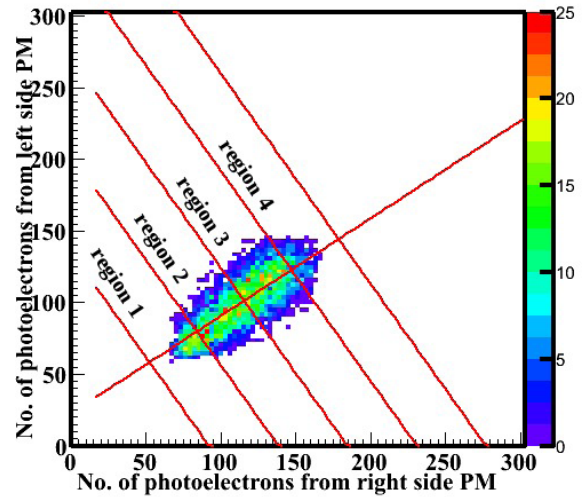


Fig. 1. Distribution of number of photoelectrons obtained from left- and right-sided photomultipliers connected to scintillator at central position.

$$(3) \quad Cov_{(i,j)}(z, \text{region}) = \frac{\sum_{k=1}^N (\vec{t}_{(k(i))}(z, \text{region}) - \vec{t}_{\text{avg}(i)}(z, \text{region})) \cdot (\vec{t}_{(k(j))}(z, \text{region}) - \vec{t}_{\text{avg}(j)}(z, \text{region}))}{N}$$

measured position, a covariance matrix has been constructed using Eq. (3), where k is an index that shows a current signal from the set of measured signals for given position and N is the total number of measured signals. $\vec{t}_{k(i)}$ indicates an i -th component of the k -th signal and $\vec{t}_{\text{avg}(i)}$ denotes i -th component of the average signal. $\bar{X}(\delta t, z, \text{region})$ given in Eq. (4) is the difference between the vectors describing the examined and model signals with an additional term δt , which is the parameter varied when minimizing the Mahalanobis distance:

$$(4) \quad \bar{X}(\delta t, z, \text{region})_{(i)} = \vec{t}_{\text{model}(i)}(z, \text{region}) - \vec{t}_{k(i)} - \delta t.$$

The reconstructed hit-position is the position of most similar signal from the library with respect to measured signal (i.e., model signal for which Mahalanobis distance computed for different bisected region is minimal³). The time of particle interaction is determined as a relative time between the measured signal and the most similar model signal from the library. This method also provides the determination of the gamma quantum TOF given by Eq. (5):

$$(5) \quad \begin{aligned} t_{\text{firstStrip}} &= \delta t_{\text{firstStrip}} \\ t_{\text{secondStrip}} &= \delta t_{\text{secondStrip}} \\ \text{TOF} &= t_{\text{secondStrip}} - t_{\text{firstStrip}} \end{aligned}$$

where $\delta t_{\text{firstStrip}}$ and $\delta t_{\text{secondStrip}}$ denote shifts in time obtained for two scintillators, for which the computed Mahalanobis distance defined in Eq. (2) is lowest.

³ The degree of similarity is represented by the Mahalanobis distance value.

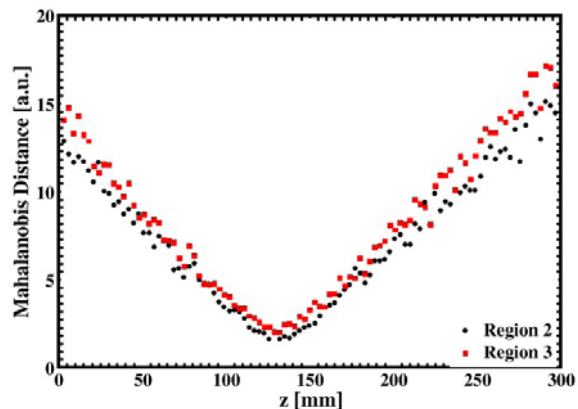


Fig. 2. Distribution of Mahalanobis distance (defined in Eq. (3)) for a signal from the central position (i.e., 150 mm) from region 2 and 3.

Preliminary results on performance of the method

An example of the Mahalanobis distance distribution calculated according to Eq. (2) for one of the measured exemplary signal at central position of the strip is shown in Fig. 2. One can see a clear

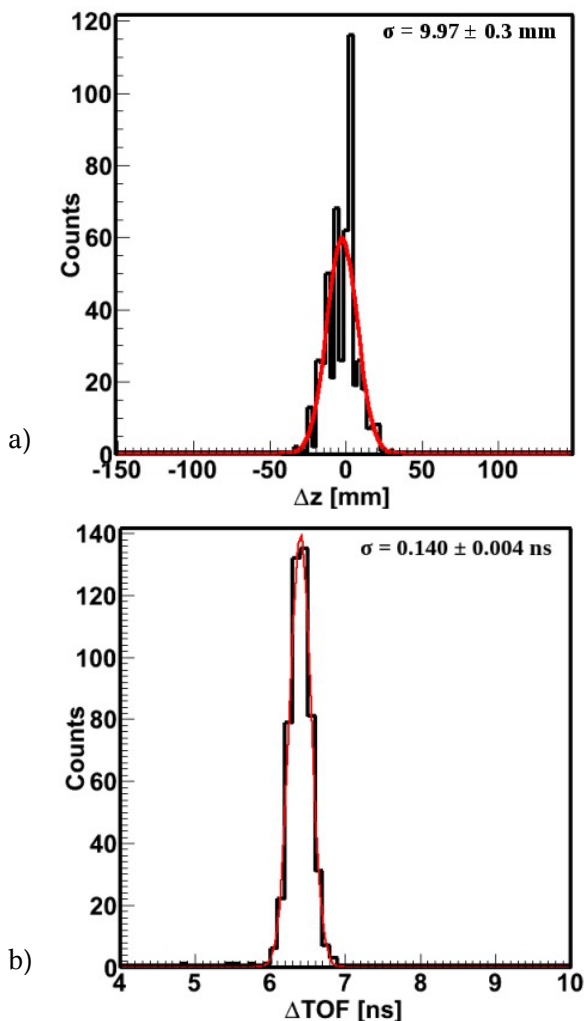


Fig. 3. (a) Distribution of differences between the true and reconstructed position Δz for signals measured at $z = 150$ mm. (b) Distribution of differences between the true and reconstructed TOF for events registered at $z = 150$ mm.

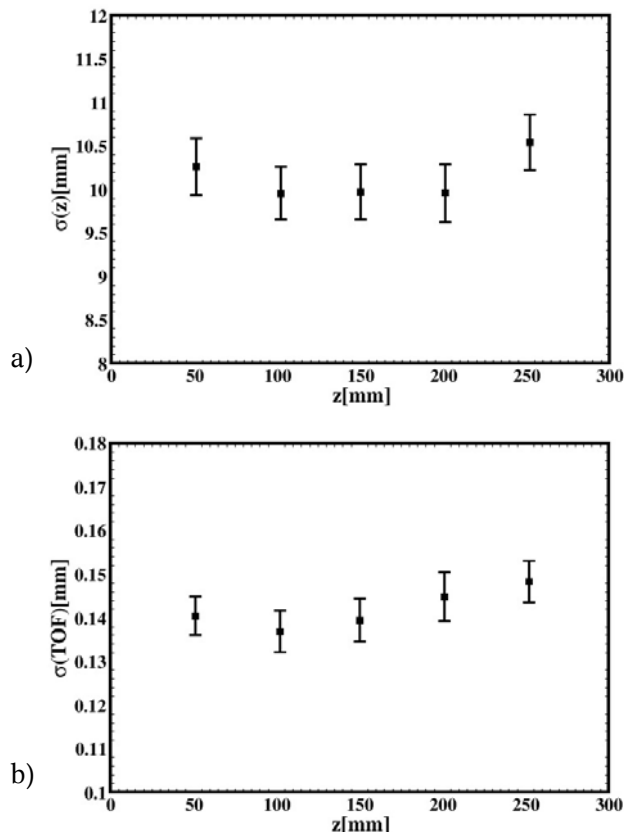


Fig. 4. (a) Position resolution as a function of the position of gamma quantum interaction. (b) TOF resolution as a function of position along the scintillator.

minimum corresponding to $z \approx 150$ mm. Figures 3a and 3b show distributions of differences between the true and reconstructed values for position and TOF, respectively⁴. The estimated resolution of position and TOF reconstruction based on these distributions amounts to $\sigma_z \approx 10$ mm and $\sigma_{TOF} \approx 140$ ps, respectively. They were determined for signals measured at several positions along the scintillator as it is shown in Figs. 4a and 4b. These results indicate that the resolutions do not change much with position.

Summary

The reconstruction method introduced in this article was validated for two-threshold levels, and the preliminary results show that it is possible to obtain a spatial resolution of about 10 mm (σ) for the gamma quanta hit position and TOF resolution of about 140 ps (σ). In the present version of the J-PET, which is being built now, we install electronics that will allow us to determine time at four-threshold levels, so a further improvement is expected in the future by including more bisected regions and more threshold levels.

⁴ In principle, the true value of TOF should be equal to zero when the source was positioned in the middle between detection modules. However, because of different electronics offsets, the reconstructed mean values of TOF may be different from zero.

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