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Thermodynamic and dynamical properties of dense ICF plasma

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Abstract. In present work, thermodynamic expressions were obtained through potentials that took into consideration long-range many-particle screening effects as well as short-range quantum-mechanical effects and radial distribution functions (RDFs). Stopping power of the projectile ions in dense, non-isothermal plasma was considered. One of the important values that describe the stopping power of the ions in plasma is the Coulomb logarithm. We investigated the stopping power of ions in inertial confinement fusion (ICF) plasma and other energetic characteristics of fuel. Calculations of ions energy losses in the plasma for different values of the temperature and plasma density were carried out. A comparison of the calculated data of ion stopping power and energy deposition with experimental and theoretical results of other authors was also performed.

Key words: Coulomb logarithm • effective interaction potential • equation of state • stopping power

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Introduction

Investigation of the interaction processes of ion beams with dense plasmas is one of the important problems in the physics of inertial confinement fusion (ICF), warm dense matter and high-power laser physics [1, 2]. Calculation of parameters of inertial fusion drivers of heavy ion beams requires adequate quantitative description of the interaction of heavy ion beams with dense plasma in a wide range of parameters. Knowledge of dynamical properties of dense plasma will enable us to calculate the design of thermonuclear target more accurately.

Experimental investigation of dense non-ideal plasmas can be based on using of a shock wave compression, a high-power laser and ion accelerator devices [3]. A characteristic feature of all of the above experiments is that the resulting plasma is non-isothermal. It is well known that the interaction potentials of particles are also of importance for correct calculation of plasma properties taking into account peculiarities and parameters of investigated plasma. These effective potentials that take into consideration long-range many-particle screening effects as well as short-range quantum-mechanical effects [4–6] are used in present work.

Interaction potentials

It is known that in order to correctly describe static and dynamic properties of plasmas, the collective

screening effect is to be taken into account. In this work the dense plasma is considered for which quantum effects must be taken into account at short distances. Further, the effective interaction potentials, which include both charge screening at large distance and quantum effects at short distance will be used [5]:

$$(1) \quad \Phi_{\alpha\beta}(r) = \frac{Z_\alpha Z_\beta}{r\sqrt{1 - \left(2\lambdabar_{\alpha\beta}/r_{\mathrm{D}}\right)^2}} \left(\exp(-rB) - \exp(-rA)\right)$$

where α, β are particle species (ion or electron), Z_α, Z_β are atomic numbers of particle species α, β, $\lambdabar_{\alpha\beta} = \hbar/\sqrt{4\pi m_{\alpha\beta} k_{\mathrm{B}} T}$ is the thermal de Broglie wavelength of pairs of particles α and β, $m_{\alpha\beta} = m_\alpha m_\beta/(m_\alpha + m_\beta)$ is the reduced mass, r_{D} is the Debye radius and

$$(2) \quad \begin{aligned} B^2 &= \frac{1}{2\lambdabar_{\alpha\beta}^2}\left(1 - \sqrt{1 - \left(\frac{2\lambdabar_{\alpha\beta}}{r_{\mathrm{D}}}\right)^2}\right) \\ A^2 &= \frac{1}{2\lambdabar_{\alpha\beta}^2}\left(1 + \sqrt{1 - \left(\frac{2\lambdabar_{\alpha\beta}}{r_{\mathrm{D}}}\right)^2}\right) \end{aligned}$$

Thermodynamic properties

Pair correlation functions were calculated on the basis of the effective interaction potentials in exponential approximation:

$$(3) \quad g_{\alpha\beta}(r) = \exp\left(-\frac{\Phi_{\alpha\beta}(r)}{k_{\mathrm{B}} T_{\alpha\beta}}\right)$$

where $\Phi_{\alpha\beta}(r)$ is the effective interaction potential of α and β type particles.

In this work, thermodynamic properties were obtained through pair correlation functions from approximation (3) and potentials (1):

$$(4) \quad P = P_{id} - \frac{2}{3}\pi \sum_{a=i,e} n_a \sum_{b=i,e} n_b \int_0^\infty \frac{\partial \Phi_{a,b}(r)}{\partial r} g_{a,b}(r) r^3 dr$$

$$(5) \quad E = E_{id} - \pi \sum_{a=i,e} n_a \sum_{b=i,e} n_b \int_0^\infty \Phi_{a,b}(r) g_{a,b}(r) r^2 dr$$

Thermodynamic expressions were used for solving the Hugoniot equation [7]:

$$(6) \quad H(V,P,E) = E - E_0 + \tfrac{1}{2}(V - V_0)(P + P_0) = 0$$

where $P_0 = 0$, $\rho_0 = 0.171$ g/cm^3, $E_0 = -15.886$ eV/atom, V_0 – volume of gas, V – volume of plasma.

If we rewrite this equation as function of the density and temperature, then we get a non-linear equation with two unknowns:

$$(7) \quad H(\rho, T) = 0$$

Shock adiabat or Hugoniot adiabat H binds the density and pressure of the plasma in front of and behind the shock front. The plasma is formed by compression, acceleration and heating of matter
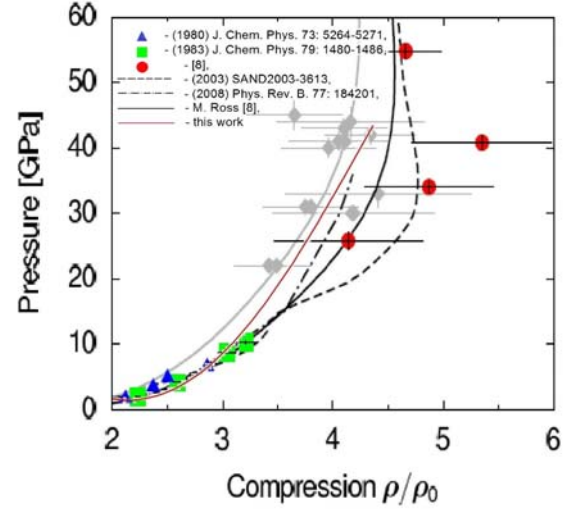

Fig. 1. Pressure of partially ionised hydrogen plasma.

in front of the shock wave. Parameters of plasma change very rapidly and in a very narrow field with the passage through the shock wave.

The pressure of partially ionised hydrogen plasma is shown on Figs. 1 and 2. In Fig. 1, blue triangles are data from Dick & Kerley, green squares are from Nellis *et al*. and red circles are from Sano *et al*. [8], red line presents the result of this work. Solid line is theoretical predictions from the model EOS of Kerley, dashed line is quantum molecular dynamics simulations and dot-dashed line is the linear mixing model. For reference, the Hugoniot data for liquid deuterium are shown by grey diamonds and grey curve is the EOS model for deuterium.

In Fig. 2, the theoretical hydrogen Hugoniot curves are shown as a solid black line (H-REOS.2), a dash green line (H-SCvH-I) and a dash blue line (Sesame-5251). Experimental data are given as grey filled squares (SNL Z-pinch), open squares (modified omega laser), circles (explosives), diamonds (gas gun). A dot-dashed orange curve (S) shows part of the Saturn adiabat [9] and a red line data from this work.
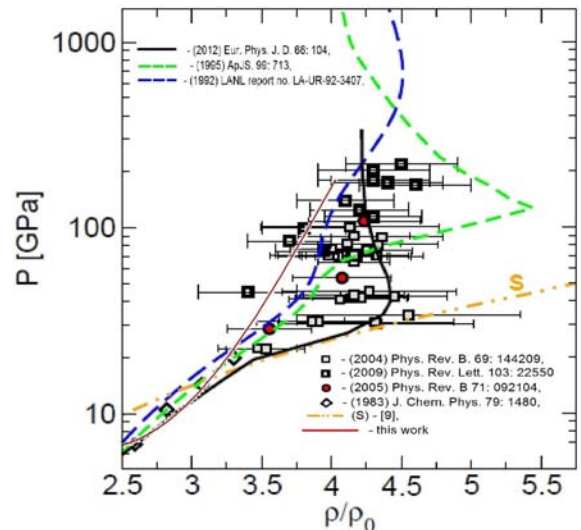

Fig. 2. Pressure of partially ionised hydrogen plasma.

Dynamical properties of dense ICF plasma

One of the most important parameters to describe the interaction of ions with matter is the energy of the projectiles. The stopping power is a parameter characterising the rate of loss of the average energy of fast electrons or ions in plasma [10, 11]:

$$
(8) \qquad \frac{dE}{dx} = 8\pi n \left(\frac{\mu_{\alpha\beta}}{m_\beta} \right) \cdot E_c \cdot b_\perp^2 \cdot \Lambda_{\alpha\beta}
$$

where $E_c = \tfrac{1}{2} m_{\alpha\beta} v^2$ is the energy of the centre of mass, v is the relative velocity of the scattered test particle; $b_\perp = (Z_\alpha Z_\beta e^2)/2E_c$, $\Lambda_{\alpha\beta}$ is the Coulomb logarithm.

The Coulomb logarithm on the basis of the effective interaction potential of the particles is determined by the scattering angle of the pair Coulomb collisions. Introducing the centre of mass in the collision process, the Coulomb logarithm reads [12, 13]:

$$
(9) \qquad \Lambda_{\alpha\beta} = \frac{1}{b_\perp^2} \int_0^{b_{max}} \sin^2\left(\frac{\theta_c}{2} \right) \cdot b \, db
$$

where $b_{max} = r_D$.

The center-of-mass scattering angle can be obtained from the known formula [12, 13]:

$$
(10) \qquad \theta_c = \pi - 2b \int_{r_0}^{\infty} \frac{dr}{r^2} \left(1 - \frac{\Phi_{\alpha\beta}(r)}{E_c} - \frac{b^2}{r^2} \right)^{-1/2}
$$

here $b_{min} = \max\{b_\perp, \lambda_{\alpha\beta}\}$ describes the minimum impact parameter.

In formula (10) $\Phi_{\alpha\beta}(r)$ is the interaction potential and r_0 is the distance of the closest approach for a given impact parameter b:

$$
(11) \qquad 1 - \frac{\Phi_{\alpha\beta}(r_0)}{E_c} - \frac{b^2}{r_0^2} = 0
$$

The basis of controlled nuclear fusion is to provide a fusion reaction of light nuclei. In this connection, the reaction involving the hydrogen isotopes deuterium and tritium (DT-cycle) is most important. The energy of the synthesis is arranged in the fuel. In case of DT reaction:

$$
(11a) \qquad D + T \rightarrow \alpha(3.5 \text{ MeV}) + n(14.1 \text{ MeV}).
$$
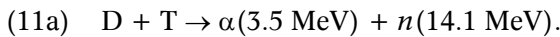
In this reaction, the total energy of 17.6 MeV is distributed between the α-particle (3.5 MeV) and the neutron (14.1 MeV). To absorb the energy of α-particle, the size of the fuel has to exceed the range ρr [14]. The range of particles is determined as follows:

$$
(12) \qquad \rho r = \int_E^{E_0} \left(\frac{dE}{\rho dx} \right)^{-1} dE
$$

where the value of the stopping power dE/dx is calculated in accordance with Eq. (8), E_0 is the initial energy of the particle. It should be noted that the range, defined as a product of mass density and a distance, is expressed in [g/cm²]. For a more accurate description of the thermonuclear fusion, such parameters as a mean deflection angle, stopping time and depth of penetration of the ions also have to be introduced. The stopping time of ion in DT plasma is determined by the following equation:

$$
(13) \qquad t_{depth} = \int_E^{E_0} \left(\frac{dE}{dt} \right)^{-1} dE
$$

The penetration depth of the ion with the initial energy E_0 can be calculated using equation:

$$
(14) \qquad \rho x = \int_E^{E_0} \langle \cos\theta \rangle \left(\frac{dE}{\rho dx} \right)^{-1} dE
$$

In the present paper, the stopping time, the mean deflection angle, the depth of penetration, the effective range of the particles with different energies generated in the DT plasma are considered. The stopping time of ion of initial energy $E_0 = 12$ MeV is shown in Fig. 3.

The results presented in Fig. 3 show that the stopping time depends on the value of the initial energy as well as on density and temperature of the fuel. But the main problem is the stopping process of ions with energies up to 12 MeV during the initial time $t_{depth} \leq 2$ ps.

Figure 4 shows the mean deflection angle of the stopped ions with initial energy $E_0 = 12$ MeV in DT plasma during the stopping process.

The deflection angle is presented as function of the residual energy after the collision process, normalised by the initial energy E_0. Figure 4 shows also that the incident ion constantly changes the direction in the target while losing its energy. Therefore,
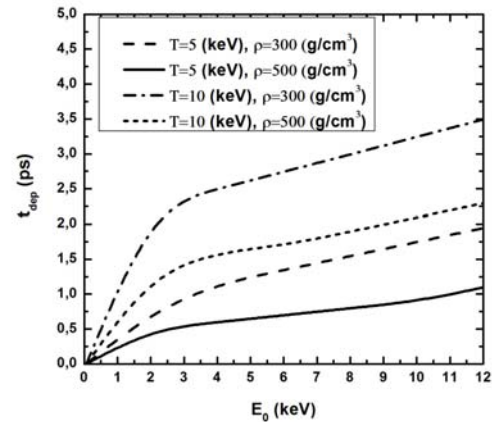

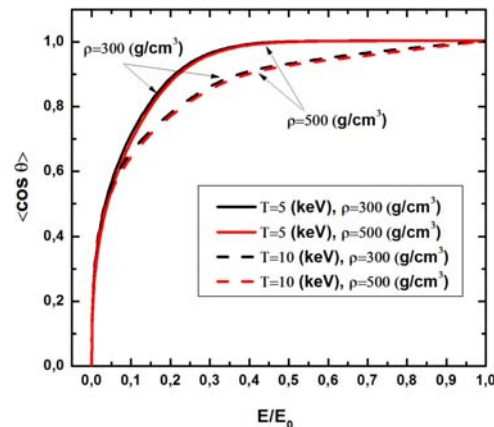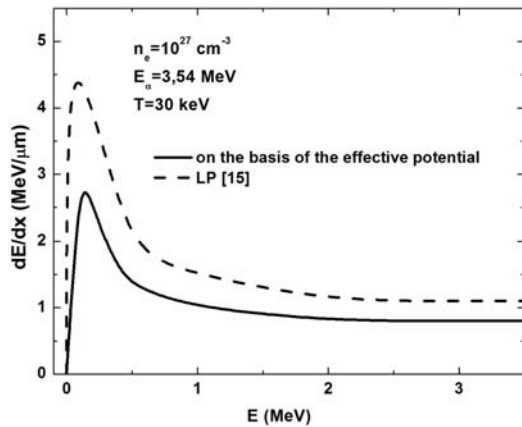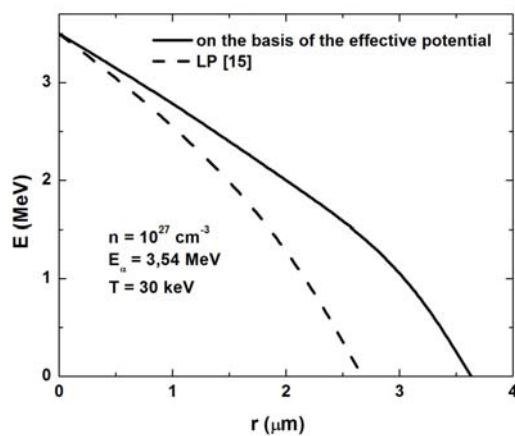**Fig. 3.** The stopping time of ions in DT plasma with initial energy $E_0 = 12$ MeV.

Fig. 4. The mean deflection angle of ions in DT plasma with initial energy $E_0 = 12$ MeV.

**Fig. 5.** The stopping power of the α-particles with an initial energy $E_0 = 3.54$ MeV.

Fig. 6. The slowing down energy of the α-particles with an initial energy $E_0 = 3.54$ MeV.

the change of the mean deflection angle with the particle density is negligible. The influence of the temperature on the mean deflection angle is much more important. The mean angle increases if the temperature increases. The values of the stopping power and slowing down of the initial energy of the α-particles under real conditions in the ICF target are presented in Figs. 5 and 6 compared with [15].

The range of α-particles with an energy of 3.5 MeV, in the plasma with a temperature of 30 keV, is about 3 g/cm². Consequently, in order to realise an efficient self-heating due to the absorption of
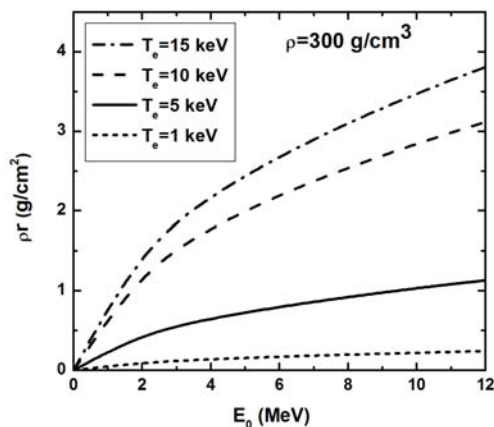
Fig. 7. The range of protons in DT plasma with a density $\rho = 300$ g/cm³.
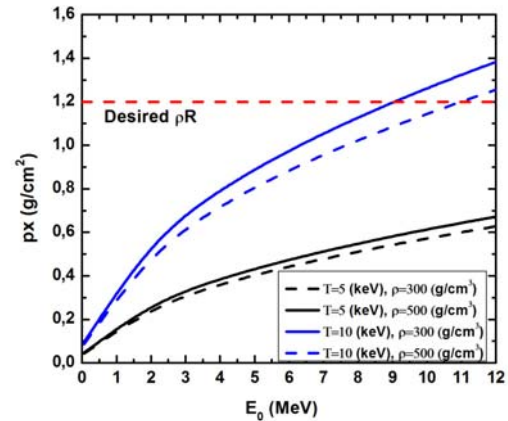
Fig. 8. The penetration depth of a proton in DT plasma at different values of density and temperature.

energy of the α-particles by the fuel, the fuel must be brought to such conditions, where $\rho r > 3$ g/cm².

Figures 7 and 8 illustrate the dependence of the range and the penetration depth of the protons with different energies on the density and temperature of the target.

The results show that at a lower target temperature of $T = 5$ keV, the target protons may conserve their energy inside the target if $\rho R < 1.2$ g/cm². However, when the target is hotter $T = 10$ keV, in order to meet the optimal deposition depth the required initial energy of the proton is reduced to $E \le 2$ MeV.

Conclusion

Thermodynamic expressions calculated on the basis of the effective interaction potential taking into account collective as well as quantum-mechanical effects were used to obtain Hugoniot adiabat. Stopping time is dependent on the values of the initial energy, density and temperature of the fuel. The influence of the temperature on the mean deflection angle is more important than that of the particle density. The protons at a temperature of $T = 5$ keV at $\rho R < 1.2$ g/cm² may conserve their energy inside the target, while for the hotter target (e.g. $T = 10$ keV), a condition of the optimal deposition depth reduces the required initial energy of the proton to $E \le 2$ MeV.

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