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Investigation of the level structure of ^{90}Nb nucleus using the shell model

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Abstract. Shell model calculations have been carried out for ^{90}Nb nucleus with the model space in which the valence protons occupy the $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ orbitals and the valence neutrons occupy the $p_{1/2}$, $g_{9/2}$, $d_{5/2}$, and $g_{7/2}$ orbitals. According to the calculated results, the negative parity is from the contribution of the proton of the $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ orbits. The moderate spin states of ^{90}Nb are mainly due to the excitation of protons from the $f_{5/2}$ and $p_{3/2}$ orbits to the $p_{1/2}$ and $g_{9/2}$ orbits across the $Z = 38$ subshell closure, and the high spin states arise from the excitation of a single neutron from the $g_{9/2}$ orbit into the $d_{5/2}$ orbit across the $N = 50$ shell closure.

Keywords: high spin state • level structure • proton excitation • shell model

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Nuclei in the $A\sim90$ mass region provide unique opportunity to investigate the influence of $Z = 38$ subshell closure and $N = 50$ shell closure on the level structures. A vast number of studies have showed that the level structures of nuclei in the $A\sim90$ mass region could be well described within the shell model framework. For example, the level structures of Sr, Y, Zr, Nb, Mo, Tc, Ru, and Rh nuclei have been well described within the shell model framework [1–11]. In those nuclei, the high spin states arise from the excitation of a single $g_{9/2}$ neutron into the $d_{5/2}$ orbit across the $N = 50$ shell closure. In particular, the $N = 49$ isotones in the $A\sim90$ mass region are of great significance for providing suitable examples for testing the residual interactions of the spherical shell model and studying the mechanism of particle--hole excitations. From this point, ^{90}Nb seemed to be an ideal candidate for such study to improve the understanding the level structures of nuclei in the $A\sim90$ mass region.

For the ^{90}Nb nucleus, semi-empirical shell model (SESM) has been used to study the level structure of ^{90}Nb in Ref. [12], but the level energies and the configurations of the 13^+_2, 15^+_1, 9^-, 17^-, and 18^- states in Fig. 1 were not provided for the absence of the level information in neighboring nuclei. Moreover, the configurations of 10^+ and 15^- states should be discussed again. In Ref. [12], the 10^+ state degenerated by the 2063 keV transition, which was suggested as the $\pi g_{9/2}\otimes\nu g_{9/2}$ configuration with a 2^+ quadrupole phonon state of ^{90}Zr core and the 15^- state degenerated by the 1904 keV transition, which was suggested as the configuration with $\pi g_{9/2}$

$\otimes\, v(f^{-1}_{5/2}g^{-1}_{9/2}g_{7/2})$. However, such gamma transitions with $E_\gamma \sim 2$ MeV in the other Nb isotopes and [90]Nb isotones at the moderate spins are usually suggested to arise from the excitation of protons from $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ orbits [6–8]. Hence, the configurations of 10^+, 11^+, and 15^- states should be associated with the excitation of protons from $f_{5/2}$, $p_{3/2}$, and $p_{1/2}$ orbits to the $g_{9/2}$ orbit. Furthermore, the high spin states of 17^- and 18^- were not discussed in Ref. [12], which could probably involved the neutron excitation across the $N = 50$ shell closure according to the systematic analysis.

In the present work, the level structure of [90]Nb is investigated by the shell model with a large configuration space and taking the neutron excitation across the $N = 50$ shell closure into consideration. The calculation adopts the NuShellx code with the GWB model space and the GWBXG residual interaction [13]. The model space utilized in the calculation consists of four proton orbits ($f_{5/2}$, $p_{3/2}$, $p_{1/2}$, $g_{9/2}$) and six neutron orbits ($p_{1/2}$, $g_{9/2}$, $d_{5/2}$, $g_{7/2}$, $d_{3/2}$, $s_{1/2}$). It takes the [66]Ni ($Z = 28$, $N = 38$) nucleus as the core. The [90]Nb nucleus has 13 valence protons and 11 valence neutrons in the considered configuration space. Considering the computational difficulties, truncations were employed in our calculation based on the analogous scheme introduced in Ref. [8]. We restrict one neutron lifted from the $g_{9/2}$ orbital into the $d_{5/2}$ or $g_{7/2}$ orbital and neglect the excitation of neutron from the $g_{9/2}$ orbital to the $s_{1/2}$ and $d_{3/2}$ orbits. The single particle energies (SPEs) corresponding to the model space are set as the same in Refs. [7, 14, 15]:

$$\varepsilon^{\pi}_{f_{5/2}} = -5.322 \text{ MeV} \quad \varepsilon^{\pi}_{p_{3/2}} = -6.144 \text{ MeV}$$
$$\varepsilon^{\pi}_{p_{1/2}} = -3.941 \text{ MeV} \quad \varepsilon^{\pi}_{g_{9/2}} = -1.250 \text{ MeV}$$
$$\varepsilon^{v}_{p_{1/2}} = -0.696 \text{ MeV} \quad \varepsilon^{\pi}_{g_{5/2}} = -2.597 \text{ MeV}$$
$$\varepsilon^{v}_{d_{5/2}} = 1.830 \text{ MeV} \quad \varepsilon^{v}_{g_{7/2}} = 5.159 \text{ MeV}$$

The comparison of the level scheme calculated by this work and that from the experiment is shown in Fig. 1. The detailed calculation results are presented in Table 1. For the positive parity states, according to the present shell model calculation, the main configurations of the 8^+ and 9^+ states are assigned

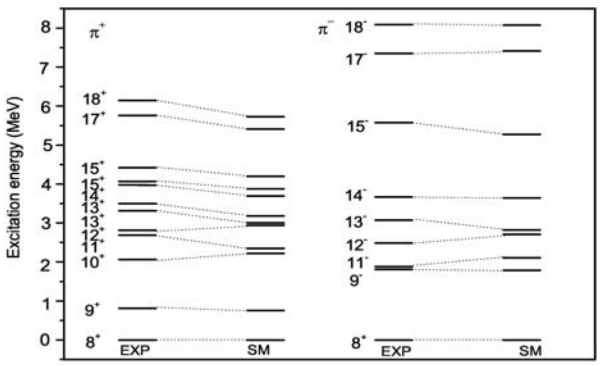

Fig. 1. Comparison of the experimental excitation energies in [90]Nb with the shell model calculations, where the experimental excitation energies are taken from Ref. [12].

Table 1. Main partitions of configurations for [90]Nb within the GWB configuration space. The configurations for a particular state would be composed of several partitions. Each partition is of the form $P = \pi[p(1), p(2), p(3), p(4)] \otimes \nu[n(1), n(2), n(3), n(4)]$, where $p(i)$ represents the number of valence protons occupying the $f_{5/2}$, $p_{3/2}$, $p_{1/2}$, and $g_{9/2}$ orbits, and $n(j)$ represents the number of valence neutrons in the ($p_{1/2}$, $g_{9/2}$, $d_{5/2}$, and $g_{7/2}$ orbits, respectively), where the experimental excitation energies are taken from Ref. [12]

$I\pi$ ħ	E_{exp} [keV]	E_{cal} [keV]	Wave function π⊗v	Partition [%]
8^+	0	0	6403⊗2900	26
			6421⊗2900	16
9^+	813	756	6403⊗2900	30
			6421⊗2900	15
10^+	2063	2219	5413⊗2900	63
			5323⊗2900	7
11^+	2689	2351	5413⊗2900	66
			5323⊗2900	9
12^+	2817	2940	5413⊗2900	51
			4423⊗2900	10
13^+_1	3314	3001	6403⊗2900	40
			4423⊗2900	19
			5413⊗2900	12
13^+_2	3496	3181	5413⊗2900	75
			5323⊗2900	10
14^+	3974	3695	6403⊗2900	39
			4423⊗2900	18
			5413⊗2900	17
15^+_1	4067	3875	5413⊗2900	78
			5323⊗2900	10
15^+_2	4420	4200	6403⊗2900	47
			4423⊗2900	16
			5413⊗2900	11
17^+	5761	5410	5413⊗2900	72
			5323⊗2900	15
18^+	6145	5725	5413⊗2900	76
			5323⊗2900	14
9^-	1809	1786	6412⊗2900	36
			5422⊗2900	24
11^-	1880	2105	6421⊗2900	72
			5422⊗2900	7
12^-	2486	2708	6412⊗2900	66
			5422⊗2900	10
13^-	3073	2821	5404⊗2900	48
			5422⊗2900	20
			4414⊗2900	9
14^-	3670	3642	5404⊗2900	46
			5422⊗2900	11
15^-	5574	5270	4414⊗2900	34
			5314⊗2900	13
			5422⊗2900	12
17^-	7348	7415	4414⊗2810	34
			5404⊗2810	21
18^-	8091	8074	5404⊗2810	29
			4414⊗2810	18
			5422⊗2900	12

as $\pi g^3_{9/2} \otimes \upsilon g^{-1}_{9/2}$ mixed with $\pi(p_{1/2} g^1_{9/2}) \otimes \upsilon g^{-1}_{9/2}$. Hence there are three valence protons occupying the $p_{1/2}$ and $g_{9/2}$ orbitals above the $Z = 38$ subshell closure for the low spin states of ^{90}Nb. The 10^+, 11^+, and 12^+ states are all degenerated by the gamma transitions with $E_\gamma \sim 2$ MeV. For the $A \sim 90$ mass region, the presence of the gamma transitions with $E_\gamma \sim 2$ MeV implies the excitation of protons across the $Z = 38$ subshell closure at moderate spin states. Indeed as the calculation results, one valence proton is excited from the $f_{5/2}$ orbitals to the $p_{1/2}$ orbital for the above three states. Hence their main configurations are assigned as $\pi(f^{-1}_{5/2} p_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$. With the increase of the excitation energy and angular momentum, the ratio of $\pi(f^{-1}_{5/2} p_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$ in the wave function significantly increases in the 13^+_1 to 18^+ states with the excitation energy of about 3 MeV to about 6 MeV. Moreover, more protons are excited across the $Z = 38$ subshell closure. The 13^+_1, 14^+, 15^+_2, 17^+, and 18^+ states have a significant portion of $\pi(f^{-1}_{5/2} p^2_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$ and $\pi(f^{-1}_{5/2} p^{-1}_{3/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$ in their wave functions. For the positive parity states of ^{90}Nb, the neutron excitation from the $g_{3/2}$ into the $d_{5/2}$ orbit across the $N = 50$ shell closure is not observed below the excitation of about 6 MeV, and all these states arise from the excitation of one or two protons from the $f_{3/2}$ and $p_{3/2}$ orbits to the $p_{1/2}$ orbits across the $Z = 38$ subshell closure.

Considering the negative parity states, the excitation energies of the 9^-, 11^-, and 12^- states is about 2 MeV. Their main configuration is $\pi(p_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$, where the $p_{1/2}$ orbit provides the negative parity. It is somehow strange that such a high excitation energy for these states is assigned as such wave functions. The reason for such a high excitation energy could be the coupling way of the last three valence protons. There are also a small contribution of one proton excitation across the $Z = 38$ subshell closure for them with the configuration of $\pi(f^{-1}_{5/2} p^2_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$. For the 13^- and 14^- states, they have the dominate one proton excitation configuration as $\pi(f^{-1}_{5/2} p^2_{1/2} g^3_{9/2}) \otimes \upsilon g^{-1}_{9/2}$ and $\pi(f^{-1}_{5/2} g^4_{9/2}) \otimes \upsilon g^{-1}_{9/2}$. One more proton is excited across the $Z = 38$ subshell closure from the $f_{3/2}$ and $p_{3/2}$ orbits to the $p_{1/2}$ and $g_{3/2}$ orbits for the 15^- state, as its excitation energy reaches 5 MeV. For the 17^- and 18^- states, besides the excitation of the proton, there is also a single neutron excited from the $g_{3/2}$ orbit into the $d_{5/2}$ orbit across the $N = 50$ closed shell and their main wave functions are $\pi(f^{-2}_{5/2} p^{-1}_{1/2} g^4_{9/2}) \otimes \upsilon g^{-1}_{9/2} d_{5/2}$ and $\pi(f^{-1}_{5/2} g^4_{9/2}) \otimes \upsilon g^{-2}_{9/2} d_{5/2}$. For all the negative parities, the negative parity comes from the contribution of the $f_{3/2}$, $p_{3/2}$, and $p_{1/2}$ proton orbits. As discussed above, the present level structure of ^{90}Nb is generated via three different mechanisms: (a) coupling of the last three valence protons from the $p_{1/2}$ and $g_{9/2}$ orbits above the $Z = 38$ subshell closure, (b) the excitation of protons from the $f_{5/2}$ and $p_{3/2}$ orbits to the $p_{1/2}$ and $g_{9/2}$ orbits across the $Z = 38$ subshell closure, and (c) the excitation of a single neutron from the $g_{9/2}$ into the $d_{5/2}$ orbit across the $N = 50$ closed shell.

In summary, the shell model calculations for the ^{90}Nb nucleus are performed using a configuration space of $\pi(f_{5/2}, p_{3/2}, p_{1/2}, g_{9/2})$ $\upsilon(p_{1/2}, g_{1/2}, d_{5/2}, g_{7/2})$. The results are compared with the experimental values, and the level structure is well reproduced. It is confirmed that the excitation of protons across the $Z = 38$ subshell closure and one neutron across the $N = 50$ closed shell are essential for the description of the level structure of ^{90}Nb. Furthermore, more experiments are needed to explore higher spin states, which investigate neutron excitations across the $N = 50$ closed shell.

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